

**Table S1 Type-I error of Wald statistics.** In this table we show the type I errors at thresholds 0.1, 0.05, 0.01, 0.005 with 5000 runs of simulation. The simulation results agree with theoretical thresholds, thus confirming that Wald statistics under the null hypothesis have correct exceedance probabilities.

Sample size	$\chi^2$ threshold			
	0.1	0.05	0.01	0.005
300	0.10	0.056	0.011	0.0064
400	0.092	0.047	0.01	0.0046
500	0.10	0.047	0.0096	0.0052
600	0.11	0.053	0.0086	0.0046