File S1

Rapid adaptation of a polygenic trait after a sudden environmental shift

Supporting Information

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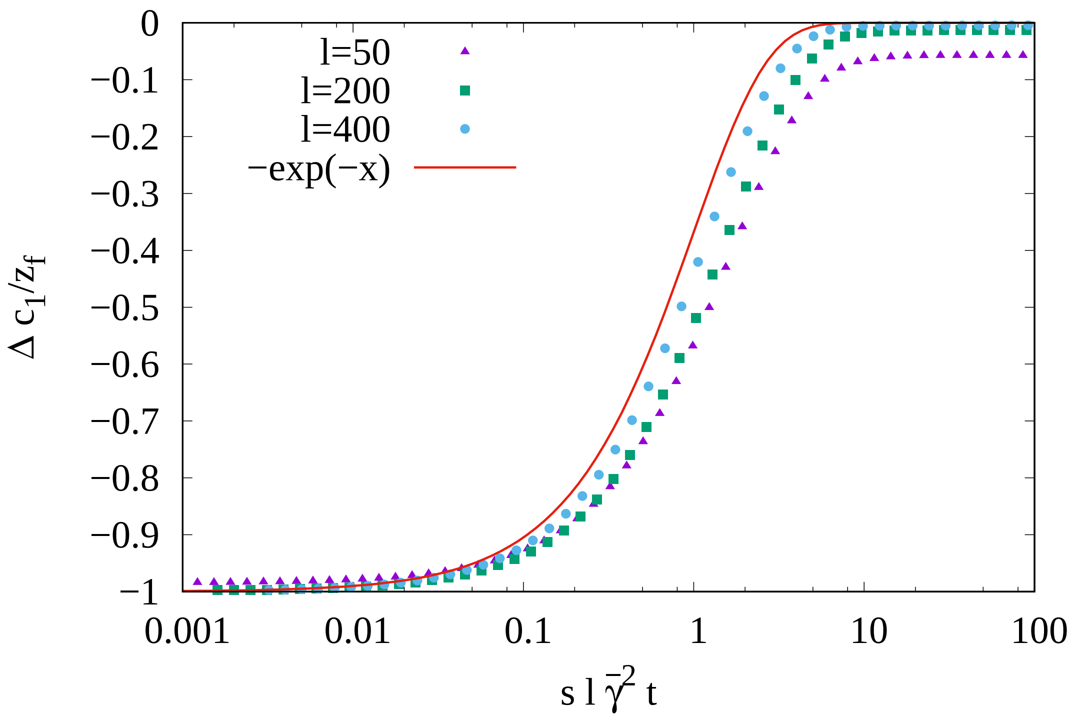

Figure S1: Mean deviation for various values of the number of loci and fixed $z_f = \ell\bar{\gamma}/4$ for the parameters in Fig. 1 to show that the numerical data obtained in the full model (6) for a single set of effects approaches the analytical expression (18) with increasing ℓ.

S1 Cumulant hierarchy

Here we briefly discuss a commonly used method and its inadequacy. The equation (13) for the nth cumulant is related to one higher one and to break this hierarchy, one may set all the cumulants higher than n^* to be zero. When most effects are small, the dynamics can be described by setting the second cumulant to be constant and all the higher cumulants to be zero ($n^* = 2$). However, when most effects are large, the standard approximation of terminating the infinite set of differential equations for the cumulants by setting $c_n(t) = 0, n > n^*$ can not capture these dynamics. As Fig. S2 shows, the essential features of the cumulants are not captured even when $n^* = 9$.

This rather unexpected result can be understood by noting that the cumulants (except mean) vary *nonmonotonically* with time as shown in Figure S2. Around $t = 50$, the variance has a maximum but the mean has not equilibrated. Then by (13) for $n = 2$, the LHS is zero at $t = 50$ and therefore the third cumulant must vanish as indeed seen in Fig. S2C. But if we were to choose $n^* = 3$ (i.e., $c_3(t) = c_3(0), c_n(t) = 0, n > 3$), the derivative of the variance can never be zero and the nonmonotonic behavior of the variance can not be captured by such an approximation. A similar argument holds for higher cumulants since they also oscillate in time.

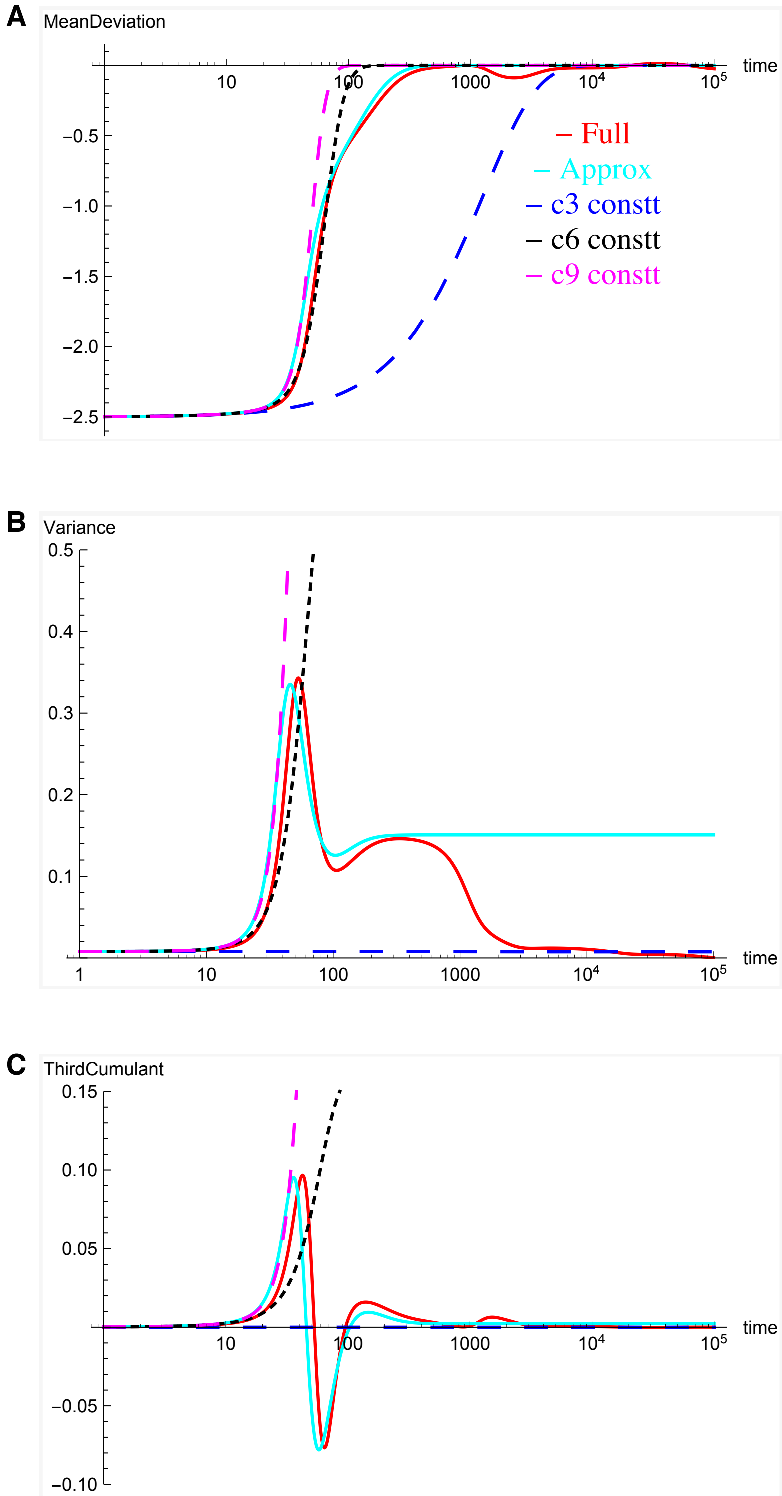

Figure S2: Dynamics of the cumulants when most effects are large: Numerical results obtained by closing the hierarchy in (13) when the third, sixth and ninth cumulants are assumed to be constant. Full and Approx refer to results obtained using the full model (6) and directional selection model (10). The parameters are the same as in Fig. 3.

α	β	Integral J_l
20	0.1	0.00646802
	0.5	0.00782825
	1	0.0132363
	2	0.108583
	3	0.313421
	4	0.506622
	5	0.650011
	6	0.74967
200	0.1	0.000120827
	0.5	0.000135405
	1	0.000201675
	2	0.00935999
	3	0.0704926
	4	0.181047
	5	0.304219
	6	0.417745
2000	0.1	1.78243×10^{-6}
	0.5	1.92926×10^{-6}
	1	2.6638×10^{-6}
	2	0.000880179
	3	0.0160675
	4	0.0635073
	5	0.137642
	6	0.223307

Table S1: Numerical evaluation of the integral J_l defined by (F.8) for various α and β.