File S4

Stability and response of polygenic traits to stabilizing selection and mutation. Supplementary Information 4

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4 Effective number of loci

The response of the trait is given by $d\bar{z}/dt = 2\nu S\Delta\Omega$. If n_e effective loci contribute to the variance, then by assuming the HoC we get that

$$\frac{d\bar{z}}{dt} = 4n_e\mu\Delta\Omega \ . \tag{19}$$

We saw that the alleles that first respond to a sudden shift in the optimum are those of effect close to $\hat{\gamma}$. Assuming no initial deviations from the optimum, and using Eq. 12 of the main text we get

$$\frac{d\bar{z}}{dt} = \Delta\Omega\left(4\mu n_f + S\sum_{k\in\mathbb{S}}\gamma_k^2\right) . \tag{20}$$

Hence, in as long as there are some fixed alleles ($n_f > 0$), the two expressions of $d\bar{z}/dt$ equate to give

$$n_e = n_f\left(1 + \frac{\frac{1}{2}\sum_{k\in\mathbb{S}}\gamma_k^2}{2\mu n_f/S}\right) . \tag{21}$$

In essence, this amounts to the variance of the HoC model but using an effective number of loci n_e.

A similar approximation using effective number of loci for the initial response of the genetic variance, indicates that ν remains roughly constant. Therefore the approximations can be regarded in a "breeder's equation regime", where the response to selection is sustained with constant genetic variance.