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(* This Mathematica 5.2 notebook contains calculations for the stability analysis
  in the article "Two-locus epistasis with sexually antagonistic selection: A
    genetic Parrondo's paradox" submitted to Genetics by Floyd A. Reed, 2007 *)

(* define the change in frequency in males *)

vbar = v xf am af + vh ( xf am (1 - af) + xf af (1 - am)) + xf (1 - am) (1 - af) +
   (1 - xf) am af + (1 - xf) am (1 - af) + (1 - xf) af (1 - am) + (1 - xf) (1 - am) (1 - af);

(* the change in x in males *)

nx = v xf am af + vh ( xf am (1 - af) + xf af (1 - am)) + xf (1 - am) (1 - af);
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In[5]:=

$$xmp = \frac{nx}{vbar};$$

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(* the change in a in males *)
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$$na = v\,xf\,am\,af + vh\left(\frac{xf\,am\,(1 - af) + xf\,af\,(1 - am)}{2}\right) + (1 - xf)\,am\,af + \frac{(1 - xf)\,am\,(1 - af) + (1 - xf)\,af\,(1 - am)}{2};$$

In[10]:=

$$amp = \frac{na}{vbar};$$

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(* define the changes in frequency in the females *)

wbar = w xm xf am af +
   wh (xm xf am (1 - af) + xm xf af (1 - am) + xm (1 - xf) am af + xm (1 - xf) am (1 - af) +
      xm (1 - xf) af (1 - am) + xf (1 - xm) am af + xf (1 - xm) am (1 - af) + xf (1 - xm) af (1 - am))
   xm xf (1 - am) (1 - af) + xm (1 - xf) (1 - am) (1 - af) + xf (1 - xm) (1 - am) (1 - af) +
   (1 - xm) (1 - xf) am af + (1 - xm) (1 - xf) am (1 - af) +
   (1 - xm) (1 - xf) af (1 - am) + (1 - xm) (1 - xf) (1 - am) (1 - af);

(* the change in x in females *)
```

$$gx = w\,xm\,xf\,am\,af + wh\left(xm\,xf\,am\,(1 - af) + xm\,xf\,af\,(1 - am) + \frac{xm\,(1 - xf)\,am\,af + xm\,(1 - xf)\,am\,(1 - af) + xm\,(1 - xf)\,af\,(1 - am)}{2} + \frac{xf\,(1 - xm)\,am\,af + xf\,(1 - xm)\,am\,(1 - af) + xf\,(1 - xm)\,af\,(1 - am)}{2}\right) + xm\,xf\,(1 - am)\,(1 - af) + \frac{xm\,(1 - xf)\,(1 - am)\,(1 - af) + xf\,(1 - xm)\,(1 - am)\,(1 - af)}{2};$$

In[16]:=

$$xfp = \frac{gx}{wbar};$$

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(* the change in a in females *)

ga = w xm xf am af + wh ( xm (1 - xf) am af + xf (1 - xm) am af +
       (xm xf am (1 - af) + xm (1 - xf) am (1 - af) + xf (1 - xm) am (1 - af)) / 2 +
       (xm xf af (1 - am) + xm (1 - xf) af (1 - am) + xf (1 - xm) af (1 - am)) / 2 ) +
    (1 - xm) (1 - xf) am af + ((1 - xm) (1 - xf) am (1 - af) + (1 - xm) (1 - xf) af (1 - am)) / 2;
```

In[20]:=
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afp = ga / wbar;
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(* set up a Jacobian matrix of the partial derivatives,
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$$J = \begin{bmatrix} \frac{\partial x'_m}{\partial x_m} & \frac{\partial x'_m}{\partial x_f} & \frac{\partial x'_m}{\partial a_m} & \frac{\partial x'_m}{\partial a_f} \\ \frac{\partial x'_f}{\partial x_m} & \frac{\partial x'_f}{\partial x_f} & \frac{\partial x'_f}{\partial a_m} & \frac{\partial x'_f}{\partial a_f} \\ \frac{\partial a'_m}{\partial x_m} & \frac{\partial a'_m}{\partial x_f} & \frac{\partial a'_m}{\partial a_m} & \frac{\partial a'_m}{\partial a_f} \\ \frac{\partial a'_f}{\partial x_m} & \frac{\partial a'_f}{\partial x_f} & \frac{\partial a'_f}{\partial a_m} & \frac{\partial a'_f}{\partial a_f} \end{bmatrix}$$

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*)

jmx = {{D[xmp, xm], D[xmp, xf], D[xmp, am], D[xmp, af]},
   {D[xfp, xm], D[xfp, xf], D[xfp, am], D[xfp, af]},
   {D[amp, xm], D[amp, xf], D[amp, am], D[amp, af]},
   {D[afp, xm], D[afp, xf], D[afp, am], D[afp, af]}};
```

In[22]:=
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(* evaluate at the first fixation point (0,0,0,0) *)
xm = 0; xf = 0; am = 0; af = 0; Eigenvalues[jmx]
```

Out[22]= $\{1, 1, -\frac{1}{2}, 0\}$

In[23]:=
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Clear[xm, xf, am, af]
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In[24]:= (* evaluate at the last fixation point (1,1,1,1) *)
         xm = 1; xf = 1; am = 1; af = 1; Eigenvalues[jmx]
```

Out[24]= $\left\{0, \frac{vh\, w + v\, wh}{2\, v\, w}, \frac{v\, wh - \sqrt{v}\, \sqrt{wh}\, \sqrt{8\, w + v\, wh}}{4\, v\, w}, \frac{v\, wh + \sqrt{v}\, \sqrt{wh}\, \sqrt{8\, w + v\, wh}}{4\, v\, w}\right\}$

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In[25]:= Clear[xm, xf, am, af]

In[26]:= (* evaluate at the fixation point (1,1,0,0) *)
         xm = 1; xf = 1; am = 0; af = 0; Eigenvalues[jmx]
```

Out[26]= $\left\{1, -\frac{1}{2}, 0, \frac{vh + wh}{2}\right\}$

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In[27]:= Clear[xm, xf, am, af]

In[28]:= (* evaluate at the fixation point (0,0,1,1) *)
         xm = 0; xf = 0; am = 1; af = 1; Eigenvalues[jmx]
```

Out[28]= $\left\{1, 0, \frac{1}{4}\left(wh - \sqrt{wh}\, \sqrt{8\, v + wh}\right), \frac{1}{4}\left(wh + \sqrt{wh}\, \sqrt{8\, v + wh}\right)\right\}$