Instructions for matrix_builder.nb

For arbitrary numbers of incompatibility factors, Mathematica notebook matrix_builder.nb computes the transmission matrix $C$ for a single neutral marker and the right eigenvector $q$ associated with the leading eigenvalue. It accommodates autosomal- and X-linkage of the neutral marker and of the incompatibility factors. It assumes no functional epistasis among the incompatibility factors in the determination of fitness. No linkage occurs among the factors, but the marker may be linked to one of the factors.

Specification of parameters is done in the section “Variables Settings”:

- $nloci$: number of incompatibility factors
- $locpos$: location of each incompatibility factor
  
  Use $a$ to specify the autosomal factors, followed by $X$ for the X-linked factors.
- $markerpos$: location of the marker
  
  Use $a$ for an autosomal and $X$ for an X-linked marker.
- $deltaf$: fitness parameters for female carriers of the factors
  
  Ordering of the factors must adhere to $locpos$.
- $deltam$: fitness parameters for male carriers of the factors
- $recf$: rate of crossing-over between the marker and each factor in females
  
  The marker can be linked to at most one factor. For all unlinked factors, specify 1/2.
- $recfm$: rate of crossing-over between the marker and each factor in males
  
  As males are assumed to be hemizygous for the X chromosome, an absence of crossing-over is assumed for X-linked factors and crossover rates are not specified. For other chromosomes, enter crossover rates as for $recf$.

Select “Evaluate Notebook” under the “Evaluate” menu to execute the script. Variable $c$ contains the transmission matrix and variable $q$ the right eigenvector $q$. The backgrounds on which the neutral marker can occur are indexed by $i = 1, \ldots, 2^{nloci}$, with the first $2^{nloci}$ backgrounds corresponding to female carriers and the remaining to male carriers. If $i \leq 2^{nloci}$ (female carrier), background $i$ is given by the binary representation of $2^{nloci} - i$; if $i > 2^{nloci}$ (male carrier), it is given the binary representation of $2 \cdot 2^{nloci} - i$. 

D.Fusco and M.K.Uyenoyama
Example  Consider an X-linked marker together with incompatibility loci 1 (autosomal) and 2 (X-linked). Crossing-over between the marker and locus 1 occurs at rate 1/2 in both sexes and between the marker and locus 2 at rate $r_f$ in females and $r_m$ in males. Each foreign allele at locus $i$ ($i = 1, 2$) reduces the fitness of a female carrier by a factor of $\sigma_{f,i}$ and the fitness of a male carrier by a factor of $\sigma_{m,i}$.

In the “Variables Settings” section, we set:

- $nloci=2$
- $locpos=\{a,X\}$
- $markerpos=X$
- $deltaf=\{\sigma_{f,1}, \sigma_{f,2}\}$
- $deltam=\{\sigma_{m,1}, \sigma_{m,2}\}$
- $recf=\{1/2, r_f\}$
- $recm=\{1/2\}$

The order of the states in the matrix $c$ and in the vector $q$ follows the index given by the following table (1 means present, 0 means absent):

<table>
<thead>
<tr>
<th>Index</th>
<th>Carrier</th>
<th>Factor 1</th>
<th>Factor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Female</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Female</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Female</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Female</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Male</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Male</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Male</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>Male</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Eigenvector $q$ corresponds to

$$q = \left( \frac{1 - f}{f} \eta_{f,12}, \frac{1 - f}{f} \eta_{f,1}, \frac{1 - f}{f} \eta_{f,2}, \frac{1 - f}{f} \eta_{m,12}, \eta_{m,1}, \eta_{m,2}, 1 \right).$$

For a female migrant, the reproductive value at the marker is given by

$$\omega = \frac{(f, 0, 0, 0, 1 - f, 0, 0, 0)q}{(0, 0, 0, f, 0, 0, 0, 1 - f)q} = \frac{(\eta_{f,12} + \eta_{m,12})}{2},$$

2 SI D.Fusco and M.K.Uyenoyama
in which $f$ is the proportion of females among offspring. For a male migrant, the reproductive value at the marker corresponds to

$$\omega = \frac{(f, 0, 0, 0, 0, 0, 0, 0)q}{(0, f, 0, 0, 0, 0, 0, 0)q} = \frac{f_1q(1)}{f_4q(4)} = \eta_{f,12},$$

reflecting that the male migrant transmit its X-linked marker only to its daughters.