File S2

Proof of Result 2

If an asymmetric polymorphism exists, then (11) holds, namely, (with $\mu = \mu_B$),

$$1 + sy = \frac{(1 - 2\mu)(1 + s)}{1 + sx}. \quad (S2.1)$$

That is,

$$y = \frac{s(1 - x) - 2\mu(1 + s)}{s(1 + sx)}, \quad 1 - y = \frac{s(1 + s)x + 2\mu(1 + s)}{s(1 + sx)}. \quad (S2.2)$$

Substituting these relations into the equilibrium equation for $x$ from (8), we find, after some simplification, that

$$x = \frac{1 - m}{1 + sx}[(1 - \mu)(1 + s)x + \mu(1 - x)] + \frac{m}{s}(sx + 2\mu + \mu s). \quad (S2.3)$$

Equation (S2.3) is equivalent to the quadratic equation

$$T(x) = (1 - m)s^2 x^2 - sx[s(1 - m) - \mu(s + 2)(1 - 2m)] - \mu(2m + s) = 0. \quad (S2.4)$$

As $\mu, m, s$ are positive and $m < 1$, we have $T(0) < 0$ and $T(\pm \infty) > 0$, implying that $T(x)$ has two real roots, one positive and one negative. Now

$$T(1) = (1 - m)s^2 - s[s(1 - m) - \mu(s + 2)(1 - 2\mu)] - \mu(2m + s)$$

$$= \mu[s(s + 2)(1 - 2m) - (2m + s)]. \quad (S2.5)$$

$T(1; m)$ is a linear function of $m$ and

$$T(1; 0) = \mu s(s + 1) > 0$$

$$T(1; \frac{1}{2}) = -\mu(2m + s) < 0 \quad (S2.6)$$

$$T(1; m_0) = 0.$$ 

Hence if $0 < m < m_0$, $T(1; m) > 0$ and a unique $0 < \hat{x} < 1$ exists such that $T(\hat{x}) = 0$. In order for $\hat{x}$ to be an equilibrium, its corresponding $\hat{y}$ should satisfy $0 < \hat{y} < 1$, where

$$1 - \hat{y} = \frac{1 + s}{1 + s\hat{x}} \frac{s\hat{x} + 2\mu}{s} \quad (S2.7)$$

and $0 < \hat{y} < 1$ if and only if

$$(1 + s)(s\hat{x} + 2\mu) < s(1 + s\hat{x}) \quad (S2.8)$$
or
\[
\hat{x} < \frac{s - 2\mu(1+s)}{s}. \tag{S2.9}
\]

So \(0 < \hat{x} < 1\) if \(0 < \mu < \mu_0 = \frac{1}{2} \frac{s}{s+1}\), and \([s - 2\mu(1+s)] > 0\). We compute \(T\left(\frac{s-2\mu(1+s)}{s}\right)\), which equals
\[
(1-m)[s-2\mu(1+s)]^2 - [s-2\mu(1+s)]\left[s(1-m) - \mu(s+2)(1-2m)\right] - \mu(2m+s). \tag{S2.10}
\]

So
\[
T\left(\frac{s-2\mu(1+s)}{s}\right) = 2\mu^2(1+s)(s+2m) + s\mu(s+2)(1-2m) - \mu(2m+s). \tag{S2.11}
\]

But when \(0 < m < m_0\),
\[
T(1) = s\mu(s+2)(1-2m) - \mu(2m+s) > 0, \tag{S2.12}
\]

therefore \(T\left(\frac{s-2\mu(1+s)}{s}\right) > 0\), and (S2.9) holds.