File S3

Proof of Result 3

The asymmetric equilibrium \((\hat{x}, \hat{y})\) is determined by the solutions of the quadratic equation

\[
T(x; \mu_B) = (1 - m)s^2x^2 - sx[s(1 - m) - \mu_B(s + 2)(1 - 2m)] - \mu_B(2m + s) = 0. \tag{S3.1}
\]

When \(\mu_B = 0\), equation (S3.1) reduces to

\[
T(x; 0) = -(1 - m)s^2x(1 - x) = 0, \tag{S3.2}
\]

giving the two solutions \(\hat{x} = 0\) and \(\hat{x} = 1\). As \(T(0; \mu_B) < 0\), when \(\mu_B > 0\) the solution \(x = 0\) shifts to a negative solution of (S3.1). Hence, when \(\mu_B\) is positive and small, the positive root \(\hat{x}(\mu_B)\) of \(T(x; \mu_B) = 0\) is close to \(x = 1\). That is, when \(\mu_B\) is small the corresponding asymmetric equilibrium is close to the fixation of \(AB\) where \(\hat{x} = \hat{y} = 1\).

Moreover, by continuity, if \(\mu_B\) is small, their stability is the same. Near fixation of \(AB\), \(w = 1 - x\) and \(z = 1 - y\) are small, and up to non-linear terms, when \(\mu_B = 0\), we have

\[
w' = \frac{1 - m}{1 + s}w + m(s + 1)z \\
z' = \frac{m}{1 + s}w + (1 - m)(s + 1)z. \tag{S3.3}
\]

The characteristic polynomial \(P(\lambda)\) of (S3.3) is

\[
P(\lambda) = \lambda^2 - (1 - m)\left[(1 + s) + \frac{1}{1 + s}\right]\lambda + (1 - 2m) \tag{S3.4}
\]

and

\[
P(1) = 1 - (1 - m)\left(\frac{(1 + s)^2 + 1}{1 + s}\right) + 1 - 2m. \tag{S3.5}
\]

In fact, it can be easily seen that

\[
(1 + s)P(1) = -s^2(1 - m). \tag{S3.6}
\]

As \(P(+\infty) > 0\) and \(P(1) < 0\), since \(s > 0\), \(0 < m < 1\), \(P(\lambda)\) has a root larger than 1. Thus, when \(\mu_B\) is small, fixation in \(AB\) is internally locally unstable and so is the asymmetric equilibrium when \(\mu_B\) is small.