Figure S1  Histogram of q estimates
Luo et al.’s equations (indicated here by superscript L) from Table 1 in (Luo et al. 2004) are

\[
\begin{align*}
p_1^1 &= \alpha (1 - r)^2 \\
p_2^1 &= \alpha \frac{r^2}{3} \\
p_1^2 &= 2 \alpha r (1 - r) \\
p_2^2 &= 2 \alpha \frac{r^2}{3} \\
p_1^3 &= \frac{2(1 - \alpha) r (1 - r)}{3} \\
p_2^3 &= \frac{2(1 - \alpha) r^2}{9} \\
p_1^4 &= (1 - \alpha) (1 - r)^2 \\
p_2^4 &= \frac{4(1 - \alpha) r (1 - r)}{3} \\
p_1^5 &= \frac{(1 - \alpha) r^2}{9} \\
p_{10}^3 &= \frac{4(1 - \alpha) r^2}{9} \\
p_{11}^3 &= \frac{2(1 - \alpha) r^2}{9}
\end{align*}
\]

Probabilities for gamete marker classes (red-only, green-only, yellow/both, brown/none) can be defined by summing up the probabilities of gamete modes, weighted by the frequencies of the class in question among all genotypes of the respective mode (see also ref. 19). For each mode, the weight is the ratio of the number of instances of that gamete mode that are compatible with the class in question (red-only etc.) and the overall number of instances for that mode. The latter numbers are independent of actual genotypes and can for example be taken from Table 1 in ref. 14. The former numbers depend on the genotype. In the single-copy case (genotype RG/XY/XY/XY, with XY representing wild-type chromosomes), these numbers (“multipliers”) are:

\[
\begin{align*}
M_x^5 &= (0,3,3,6,0,3,0,6,0,6,6) \\
M_f^5 &= (0,3,0,3,6,0,6,0,6,6) \\
M_s^5 &= (1,0,3,0,3,6,3,6,0)
\end{align*}
\]
which defines probabilities of color classes (the single-copy case indicated by the superscript S) as:

\[
M^S = (3,6,6,3,6,3,6,3,6,3,0)
\]

From Eq. 1 in ref. 14 we obtain

\[
\begin{align*}
p^L_S &= \frac{0 p^{1L}_1 + 3 p^{1L}_2 + 3 p^{1L}_3 + 6 p^{1L}_4 + 0 p^{1L}_5 + 3 p^{1L}_6 + 0 p^{1L}_7 + 6 p^{1L}_8}{12} + \frac{0 p^{1L}_9 + 6 p^{1L}_{10} + 6 p^{1L}_{11}}{24} + 0 \\
p^L_T &= \frac{0 p^{1L}_1 + 3 p^{1L}_2 + 0 p^{1L}_3 + 3 p^{1L}_4 + 3 p^{1L}_5 + 6 p^{1L}_6 + 0 p^{1L}_7 + 6 p^{1L}_8}{12} + \frac{0 p^{1L}_9 + 6 p^{1L}_{10} + 6 p^{1L}_{11}}{24} + 0 \\
p^L_Y &= \frac{0 p^{1L}_1 + 0 p^{1L}_2 + 3 p^{1L}_3 + 3 p^{1L}_4 + 0 p^{1L}_5 + 3 p^{1L}_6 + 0 p^{1L}_7 + 6 p^{1L}_8}{12} + \frac{0 p^{1L}_9 + 6 p^{1L}_{10} + 6 p^{1L}_{11}}{24} + 0 \\
p^L_B &= \frac{3 p^{1L}_1 + 6 p^{1L}_2 + 0 p^{1L}_3 + 6 p^{1L}_4 + 3 p^{1L}_5 + 3 p^{1L}_6 + 3 p^{1L}_7 + 6 p^{1L}_8}{12} + \frac{3 p^{1L}_9 + 6 p^{1L}_{10} + 0 p^{1L}_{11}}{24} + 0
\end{align*}
\]

which simplify to

\[
\begin{align*}
p^L_S &= -\frac{(2 + \alpha)(-6 + r)r}{36} \\
p^L_T &= \frac{r (6 - 2 r + \alpha (-6 + 5 r))}{12} \\
p^L_Y &= \frac{\alpha (-3 + 6 r - 5 r^2) + 2 (3 - 3 r + r^2)}{12} \\
p^L_B &= \frac{(2 + \alpha)(-3 + r)^2}{36}
\end{align*}
\]

With \( r = (3 (-1 + 4 \alpha + \sqrt{((-1 + 4 \alpha)(-1 + 4 \beta))}))/(-4 + 16 \alpha) \) from Eq. 1 in ref. 14 we obtain the following equations for seed-count differences (assuming a backcross with wildtype plants, such that seed fluorescence probabilities are the same as gamete probabilities):

\[
\begin{align*}
p^L_S - p^L_T &= \frac{\alpha - \beta}{4} \\
p^L_B - p^L_Y &= \frac{\alpha + \beta}{4}
\end{align*}
\]

which are the same as those derived for the model presented in this article (see main text).

In the double-copy case (genotype RG/RY/XY/XY) we have the following multipliers:

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leading to these gamete color frequencies (the superscript D indicates the double-copy case), specialized from the Luo et al. model (indicated again by the superscript L):

\[
\begin{align*}
M_D^g &= (0,2,2,4,0,1,0,2,0,2,1) \\
M_D^F &= (0,2,0,1,2,4,0,2,0,2,1) \\
M_D^Y &= (1,1,3,2,3,2,3,8,3,8,4) \\
M_D^B &= (2,2,2,0,2,0,1,0,1,0,0)
\end{align*}
\]

With the above definition of \( r \) again, we obtain the following seed-count differences in the double-copy case, again assuming a backcross with wildtype plants, such that seed fluorescence probabilities are the same as gamete probabilities:

\[
\begin{align*}
p_{g}^{L_D} - p_{r}^{L_D} &= -\frac{r (-14 + \alpha (-22 + r) + 5 r)}{63} \\
p_{r}^{L_D} &= \frac{r (26 - 11 r + \alpha (-26 + 23 r))}{63} \\
p_{y}^{L_D} &= \frac{189 - 82 r + 42 r^2 + \alpha(-105 + 130 r - 114 r^2)}{252} \\
p_{b}^{L_D} &= \frac{63 - 78 r + 22 r^2 + \alpha(105 - 114 r + 26 r^2)}{252}
\end{align*}
\]

These equations are different from equations derived from the model presented in this article which are:

\[
\begin{align*}
p_{g}^{D_e} - p_{r}^{D_e} &= \frac{\alpha - \beta}{3} \\
p_{b}^{D_e} - p_{y}^{D_e} &= \frac{\alpha + \beta - 2}{3}
\end{align*}
\]
where $p_{D}^{J}$, $p_{T}^{J}$, $p_{D}^{I}$, and $p_{D}^{E}$ can be derived from $p_{I}$ to $p_{I}$, the double-copy multiplicators $M_{S}^{I}$, $M_{T}^{I}$, $M_{T}^{I}$, and $M_{S}^{I}$, and the gamete mode frequencies (see derivation of $p_{S}^{I}$ etc. above), or directly by sorting gametes according to color in the computational procedure as:

$$p_{D}^{I} = \frac{r^2}{6} + \beta \left( -\frac{1}{6} + \frac{5r}{12} + \frac{r'}{3} \right)$$

$$+ \alpha \left( \frac{1}{6} - \frac{7r}{4} + \frac{r}{3} - \frac{10r^2}{3} - \frac{3r^2}{2} - \frac{5r^3}{3} + \frac{5r^3}{6q} + \frac{4r'}{3} - \frac{r'}{3} - \frac{4rr'}{3q} + \frac{2rr'}{3q} \right)$$

$$p_{E}^{I} = \frac{r^2}{6} + \beta \left( -\frac{1}{2} + \frac{5r}{12} + \frac{r'}{3} \right)$$

$$+ \alpha \left( \frac{1}{2} - \frac{7r}{4} + \frac{r}{3} - \frac{10r^2}{3} - \frac{3r^2}{2} - \frac{5r^3}{3} + \frac{5r^3}{6q} + \frac{4r'}{3} - \frac{r'}{3} - \frac{4rr'}{3q} + \frac{2rr'}{3q} \right)$$

$$p_{E}^{I} = \frac{5}{6} - \frac{r^2}{6} + \beta \left( \frac{1}{6} - \frac{5r}{12} - \frac{r'}{3} \right)$$

$$+ \alpha \left( -\frac{1}{2} + \frac{7r}{4} - \frac{r}{3} + \frac{10r^2}{3} + \frac{3r^2}{2} + \frac{5r^3}{3} - \frac{5r^3}{6q} + \frac{4r'}{3} + \frac{2r'}{3q} + \frac{r'}{3} - \frac{4rr'}{3q} - \frac{2rr'}{3q} \right)$$

$$p_{E}^{I} = \frac{1}{6} - \frac{r^2}{6} + \beta \left( \frac{1}{2} - \frac{5r}{12} - \frac{r'}{3} \right)$$

$$+ \alpha \left( -\frac{1}{6} + \frac{7r}{4} - \frac{r}{3} + \frac{10r^2}{3} + \frac{3r^2}{2} + \frac{5r^3}{3} - \frac{5r^3}{6q} + \frac{2r'}{3q} + \frac{r'}{3} - \frac{4rr'}{3q} - \frac{2rr'}{3q} \right)$$

For the triple-copy case (genotype GR/GR/GR/XY) we have the following multipliers:

$$M_{S}^{T} = (0,1,1,0,0,0,0,0,0,0,0,0)$$

$$M_{T}^{T} = (0,1,0,0,1,0,0,0,0,0)$$

$$M_{T}^{T} = (1,0,0,0,0,0,0,0,0,0)$$

$$M_{S}^{T} = (1,1,2,2,2,2,4,2,2,4,2,2)$$

$$M_{T}^{T} = (1,1,2,2,2,2,4,2,2,4,2,2)$$

With these, we obtain from Luo et al.’s model:

$$p_{b}^{T} - p_{h}^{T} = \frac{\alpha - \beta}{3}$$

$$p_{b}^{T} - p_{h}^{T} = \frac{34 \alpha^2 - 8 (-3 + \beta) + \alpha (-109 + 50 \beta - 3 \sqrt{(-1 + 4 \alpha) (-1 + 4 \beta)})}{-24 + 96 \alpha}$$

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and from the model presented in this article:

\[ p_{g}^{Re} - p_{i}^{Re} = \frac{\alpha - \beta}{4} \]

\[ p_{b}^{Re} - p_{y}^{Re} = \frac{\alpha + \beta - 4}{4} \]
File S2

Probabilities of the eleven gamete modes and coefficients of double reduction when a second partner switch is allowed

The superscript \( S \) indicates that a second partner switch is allowed. The occurrence of two simultaneous partner switches is modeled such that they are not in the same region, that is they have to have at least one marker between them. The eleven gamete modes:

\[
\begin{align*}
\pi^S_1 &= \frac{p_{PC}q (4q - 8r + 4r^2)\tau}{16} + \frac{p_{DP}p_{PC}q (2r - 3r^2 - 4r' + 4r')\tau}{16} \\
\pi^S_2 &= \frac{p_{PC}q r^2\tau}{4} + \frac{p_{DP}p_{PC}q (-2r - r^2 + 4r' - 4r r')\tau}{16} \\
\pi^S_3 &= -\frac{p_{PC}q r (-4 + 4r)\tau}{8} - \frac{p_{DP}p_{PC}q r (-3r + 4r')\tau}{8} \\
\pi^S_4 &= \frac{p_{DP}p_{PC} (-q r^2) + 4q r r'\tau}{8} \\
\pi^S_5 &= \frac{p_{DP}p_{PC}r (-4 + 4q + 4r - 5qr)\tau}{8} + \frac{p_{PC}r (4q - 4r - 4qr)\tau}{8} + \frac{p_{DP}r (2 - 2r')\tau}{8} \\
\pi^S_6 &= -\frac{p_{DP}p_{PC}q r^2\tau}{8} + \frac{p_{DP}r'\tau}{4} \\
\pi^S_7 &= \frac{16 - 32r + 16r^2}{16} + \frac{p_{PC} (-4q + 4r + 8qr - 4r^2 - 4qr^2)\tau}{16} \\
&\quad + \frac{p_{DP} (10r - 13r^2 - 16r' + 14r r')\tau}{16} \\
&\quad + \frac{p_{DP}p_{PC} (-4r + 4r^2 + 2q r^2 + 4q r' - 6qr r')\tau}{16} \\
\pi^S_8 &= \frac{(16 - 16r)r}{8} + \frac{p_{PC} (-8 + 8r)\tau}{8} + \frac{p_{DP} (-4 + 13r - 12r')\tau}{8} \\
&\quad + \frac{p_{DP}p_{PC} (8 - 6q - 8r + 3qr + 6q r')\tau}{8} \\
\pi^S_9 &= \frac{p_{PC} (4q + 4r^2 - 4qr^2)\tau}{16} + \frac{p_{DP} (-2r + 3r^2 + 8r' - 10r r')\tau}{16} \\
&\quad + \frac{p_{DP}p_{PC} (-4r + 6qr - 4r^2 + 2q r^2 - 8qr' + 6q r r')\tau}{16} \\
\pi^S_{10} &= -\frac{3p_{DP} (r^2 - 4r r')\tau}{8} - \frac{3p_{DP}p_{PC} (-q r^2) + 2q r r'\tau}{8} \\
\pi^S_{11} &= r^2 - \frac{p_{PC} r^2\tau}{2} + \frac{p_{DP}p_{PC} (-q r) + 4r^2 - 2q r^2 + 2q r r'\tau}{8} + \frac{p_{DP} (-2r - 5r^2 + 4r' - 2r r')\tau}{8}
\end{align*}
\]

Coefficients of double reduction:
\[ a^s = \frac{p_{PC} q r}{4} \]

\[ \beta^s = \left( \frac{p_{DP} r + p_{PC} (2 (-1 + p_{DP})(-1 + r) r - q (-1 + 2 r)(1 + 2 (-1 + p_{DP}) r))}{4} \right) r \]
Superscript $S$ indicates that a second partner switch is allowed; superscript $e$ indicates that $p_{DP}$ and $p_{PC}$ are eliminated.

\[
 p_{1S}^e = \frac{a \left( (-2 + 4 q) r^4 - 4 q r' + r^2 (-20 + 21 q + 16 r' - 32 q r' + 2 r^3 (7 - 10 q - 4 r' + 8 q r')) \right)}{r \left( 8 a(1 - r + q (-1 + 2 r)) - q \tau \right)} + q \left( \beta (2 r + 4 r' - 4 r r' - (-1 + r) r \tau) \right)
\]

\[
 p_{2S}^e = \frac{-2 a \left( (-2 + 4 q) r^3 + 4 q r' + r (-8 + 5 q + 8 r' - 16 q r') + 2 r^2 (5 - 6 q - 4 r' + 8 q r')) \right)}{8 a(1 - r + q (-1 + 2 r)) - q \tau}
\]

\[
 p_{3S}^e = \frac{-2 a \left( (-2 + 4 q) r^2 + 2 (-1 + r) r \right) (r - 4 r')}{8 a(1 - r + q (-1 + 2 r)) - q \tau}
\]

\[
 p_{4S}^e = \frac{a^2 \left( q (8 - 26 r + 24 r^2 - 8 \tau) + 4 (-2 + 6 r - 5 r^2 + \tau^3) \right) + \beta q (-1 + r) r + a \left( 2 \beta (4 - 4 r + q (-4 + 5 r)) + 2 (-1 + r) r \tau - q (-1 + r (2 - 4 r') + r' + r^2 (-2 + 4 r')) \right)}{8 a(1 - r + q (-1 + 2 r)) - q \tau}
\]

\[
 p_{5S}^e = \frac{\beta q - a (q (1 - 2 r)^2 - 2 (-1 + r) r)}{8 a(1 - r + q (-1 + 2 r)) - q \tau}
\]
\[ p^S_e \]

\[-8 \alpha^2 \left( (-2 + 4 q) r^4 - 2 q r' + r^2 (-18 + 15 q + 10 r' - 20 q r') + 2 r^3 (7 - 3 r' + q (-8 + 6 r')) \right) \]

\[ + q \left( (8 + 13 \beta) r^2 - 4 r^3 + 16 \beta r' - 2 r (2 + \beta (5 + 7 r')) \right) \tau \]

\[ + a \left( -16 q r' (\beta + \tau) - 2 (-1 + 2 q) r^4 (-16 + 13 \tau) + 2 r (16 + 4 \beta (2 + 3 q r') - 16 r' \tau + q (-16 + 7 \tau + 39 r') \right) \]

\[ - r^2 (96 + 8 \beta (2 + q) - 16 \tau - 60 \tau r + q (-128 + 61 \tau + 120 r') + 2 r^3 (48 - 7 (3 + 2 r') \tau) \right) \]

\[ = \frac{4 r (8 \alpha (1 - r + q (-1 + 2 r)) - q \tau) \right) \]

\[ p^S_e = \frac{4 \alpha^2 \left( 3 (1 - 2 r)^2 (-2 + r + 2 r') - 2 (-1 + r)(4 + 3 r^2 + 2 r (-7 + 3 r')) \right) \]

\[ + q \left( 4 (-1 + r) r + \beta (4 - 13 r + 12 r')) \tau \right) \]

\[ + a \left( - \frac{4 \beta (8 - 8 r + 3 (2 + r + 2 r')) + 4 q (1 + 3 r') \tau - 2 (-1 + 2 q)r^2 (-16 + 13 \tau) \right) \]

\[ + r (q (32 - 29 \tau - 48 r' \tau) + 8 (-4 + 3 r' \tau) + r^2 (64 - 26 \tau - 24 r' + q (-96 + 68 \tau + 48 r') \tau) \right) \]

\[ = \frac{2 (8 \alpha (1 - r + q (-1 + 2 r)) - q \tau) \right) \]

\[ p^S_e = \frac{-8 \alpha^2 \left( (-2 + 4 q) r^4 + 4 q r' + r^2 (-8 + 11 q - 14 r' + 28 q r') - 2 r^3 (-2 - 3 r' + 6 q (1 + r')) \right) \]

\[ + q \left( 2 + 8 r' - q (3 + 19 r') \right) \tau \]

\[ + a \left( -8 \beta \left(-2 + q)r^2 - 4 qr' + r (-2 + 3 q (1 + r')) \right) \]

\[ + \left( + q (12 r^4 + 8 r' - 8 r^3 (2 + 5 r') - 2 r (1 + 21 r') + r^2 (11 + 72 r')) \tau \right) \]

\[ = \frac{4 r (8 \alpha (1 - r + q (-1 + 2 r)) - q \tau) \right) \]

\[ p^S_e = \frac{3 (-\beta q) + a (q (1 - 2 r)^2 - 2 (-1 + r) r')) (4 a r - 8 a r' - r \tau + 4 r' \tau) \right) \]

\[ = \frac{2 (8 \alpha (1 - r + q (-1 + 2 r)) - q \tau) \right) \]

\[ p^S_{16} = \frac{-4 \alpha^2 \left( q (1 - 2 r)^2 (r + 2 r^2 - 2 r') - 2 r (r - 5 r^2 + 2 r^3 + 2 r - 2 r') \right) \]

\[ + q \left( -2 r^3 + \beta (5 r^2 - 4 r^3 + 2 r (1 + r')) \right) \tau \]

\[ + a \left( 4 \beta (-4 r^2 + q (2 + 2 r - 2 r')) + 4 q r' - 2 r (q - 4 r' + 9 q r' \tau \right) \]

\[ = \frac{4 r (-4 + 3 q - 12 r' + 24 q r') r - 2 (-1 + 2 q)r^4 (-8 + 5 \tau) - 2 r^2 (-8 + \tau - 2 r' + q (8 - 6 \tau + 4 r' \tau) \right) \]

\[ = \frac{2 \tau (8 \alpha (1 - r + q (-1 + 2 r)) - q \tau) \right) \]

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