Peak and Persistent Excess of Genetic Diversity
Following an Abrupt Migration Increase

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Abstract

Genetic diversity is essential for population survival and adaptation to changing environments. Demographic processes (e.g., bottleneck and expansion) and spatial structure (e.g., migration, number and size of populations) are known to shape the patterns of the genetic diversity of populations. However, the impact of temporal changes in migration on genetic diversity has seldom been considered, although such events might be the norm. Indeed, during the millions of years of a species’ lifetime, repeated isolation and reconnection of populations occur. Geological and climatic events alternately isolate and reconnect habitats. We analytically document the dynamics of genetic diversity after an abrupt change in migration given the mutation rate and the number and sizes of the populations. We demonstrate that during transient dynamics, genetic diversity can reach unexpectedly high values that can be maintained over thousands of generations. We discuss the consequences of such processes for the evolution of species based on standing genetic variation and how they can affect the reconstruction of a population’s demographic and evolutionary history from genetic data. Our results also provide guidelines for the use of genetic data for the conservation of natural populations.
INTRODUCTION

Genetic diversity in a population of constant size results from the balance between the occurrence of new mutations and the loss of alleles by genetic drift (Fisher 1922; Wright 1931; Kimura and Crow 1964). The expected population genetic diversity can thus be estimated from the effective population size and the mutation rate in the population. In subdivided populations this estimate should further account for the strength of migration (Maruyama 1970; Smith 1970; Nei 1973): limited migration allows for strong differentiation between populations, while strong migration tends to homogenize genetic diversity between populations. Genetic diversity is also known to be impacted by population demographic changes; following bottlenecks and founder events, a loss of genetic diversity is expected to occur (Nei et al. 1975). Recently, spatial population expansions were shown to lead to increased differentiation between populations and to generate a low level of genetic diversity at the front of the expansion (Excoffier et al. 2009).

Although theoretical studies on the dynamics of genetic diversity in subdivided populations started appearing in the 1970s (Nei and Feldman 1972; Latter 1973; Nei 1973; Nagylaki 1974, 1977), the transient dynamics and non-equilibrium states of genetic diversity still do not have a good theoretical basis. Early authors characterized the ultimate rate of change of genetic diversity after a perturbation (either a change in population size or gene flow; Nei and Feldman 1972; Latter 1973; Nei 1973; Nagylaki 1974, 1977). They found that changes in genetic diversity are related to the total effective population size, which results in a slow dynamics of genetic diversity change. They thus first highlighted that non-equilibrium states and transient dynamics are expected to act on very large temporal scales. In particular, they showed that decreases in migration rates (population fragmentation or isolation) have long-term effects on genetic diversity: they reduce the amount of genetic diversity within populations and allow for population differentiation (Latter 1973; Takahata and Nei 1985). Additionally, it has been shown that short timescale random fluctuations in migration increase population differentiation (Nagylaki 1979; Whitlock 1992; Rice and Papadopoulos 2009) while cyclic fluctuations of gene flow (such as seasonal fluctu-
ations) mainly impact genetic diversity within populations (Karlin 1982; Shpak et al. 2010).

Although the genetic consequences of migration events (admixture) have recently received much attention (e.g., Pritchard et al. 2000; Falush et al. 2003; Price et al. 2009; Gravel 2012), their impact on genetic diversity and more particularly the expected induced transient dynamics have not received much attention.

Genetic diversity has a crucial importance in estimating populations at risk of extinction and species’ adaptive potential. Current genetic diversity characterizes species at risk of extinction through inbreeding depression, loss of genetic diversity, and accumulation of deleterious mutations (Gilpin and Soule 1986; Jimenez et al. 1994; Frankham 1995; Hedrick and Kalinowski 2000). The current level of genetic diversity (or standing genetic variation) is now widely recognized as a determinant for the adaptation of a population to a novel environment (Turner et al. 1993; Feder et al. 2003; Pelz et al. 2005; Colosimo et al. 2005; Hermisson and Pennings 2005; Myles et al. 2005; Hernandez et al. 2011; Jones et al. 2012). First, under new selective pressures, the adaptive value of a pre-existing allele can switch from neutral or deleterious to beneficial (Gibson and Dworkin 2004; Hermisson and Pennings 2005). Second, alleles from the standing genetic variation are present at higher frequencies in the population than any newly arisen (de novo) mutation are, thus they have higher fixation probabilities and lower times to fixation (Barrett and Schluter 2008). Finally, these alleles have already passed successive selective filters and are consequently more likely to be compatible with the background genome (Orr and Betancourt 2001; Schluter et al. 2004; Barrett and Schluter 2008).

Measures of genetic diversity are widely used to understand and infer the demographic and evolutionary history of populations. Indeed, statistical tests using polymorphism data can detect departure from neutrality and infer demographic or selective processes (e.g. Ewens 1972; Watterson 1978; Tajima 1983; Fu and Li 1993; Fay and Wu 2000, see review in Kreitman 2000). Furthermore, due to recent modeling advances in coalescent theory and increased genomic
data and computational power, it is now possible to distinguish different demographic scenarios (e.g. population bottleneck and subdivision, Peter et al. 2010) and estimate demographic and selective parameters (e.g. populations size and growth rate, proportion of admixture, selection coefficient) using polymorphism data (Beaumont et al. 2002; Kim and Stephan 2002; Kim and Nielsen 2004; Nielsen et al. 2005; Price et al. 2009). Nevertheless, it is often difficult to distinguish between the transient effects of demographic changes and the effects of selection on polymorphism data (Jensen et al. 2005; Nielsen 2005; Li and Stephan 2006; Kim and Gulisija 2010; Pavlidis et al. 2010). It is also difficult to distinguish between the signatures of different demographic changes such as changes in population size, number or migration rate (Wakeley 1999). A better understanding of the impact on genetic data of transient dynamics during demographic changes is necessary to disentangle these processes.

Interestingly, although the impact of population subdivision and short timescale population demographic changes on genetic diversity have received a lot of attention, other processes, such as long-term isolation and subsequent population reconnection, have received little attention. Such events have, without a doubt, occurred several times in the past, at long and short timescales. Repeated environmental changes have modified habitats and species distribution and created isolation and reconnection of populations. For example, during the climatic oscillations of the Quaternary period, temperate and tropical species were successively isolated into refugia and experienced habitat and population expansion, allowing for population reconnection (Hewitt 2000, 2004; Zhang et al. 2008; Young et al. 2009). At the same time, the reduction of sea levels (120 m lower than present, Lambeck et al. 2004) allowed the formation of land bridges that connected isolated lands in several parts of the world (Hewitt 2000). Repeated changes in water level resulted in fragmentation and fusion of basins within continents (as in the Great African Lakes, Galis and Metz 1998; Sturmbauer et al. 2001). Similarly, geological events such as volcanic eruptions induced periodic isolation and reconnection of islands (Cook 2008), while tectonic processes such as the formation of mountains isolated populations and reconnected others (Hughes and Eastwood...
cific, climatic, environmental and anthropogenic changes (e.g., global warming, urbanization and agriculture) have also played important roles in modifying the connectivity pattern between populations (Miller and Hobbs 2002; Delaney et al. 2010). Consequently, some species are currently subdivided into poorly connected or completely isolated populations; for examples ground beetles (Keller et al. 2004), salamanders (Noel et al. 2007) and crickets (Vandergast et al. 2009). In the meantime, other species experience habitat and population expansion (e.g., sparrows, white-tailed deer, zebra mussels; Waples 2010). Isolation and reconnection of populations not only reflect abiotic processes, but they can also represent spatial and temporal interactions of populations (e.g., secondary contacts; Green et al. 2010; Domingues et al. in press). Consequently, transient states of genetic diversity are expected to be the norm, and deserve much more attention.

In this study, we analytically characterized the dynamics of genetic diversity following a change in migration rate between populations, given any migration rate, mutation rate, population size and degree of fragmentation. We first analyzed how genetic diversity is affected by an event of isolation of populations, and by an event of reconnection of populations. We then generalized our results for situations where the migration rate between populations displays strong variation. We demonstrate that temporal changes of migration generate periods where genetic diversity reaches unexpectedly high values that can be maintained over thousands of generations. We also show that migration changes can produce a signature on summary statistics such as Tajima’s D and Ewens-Watterson’s statistics that cannot be differentiated from a signature of population size change or from the signature of selection. Finally, we discuss how such processes can impact observed macroevolutionary patterns of species diversity and how they can affect the reconstruction of populations’ demographic and evolutionary history from genetic data.

GENETIC DIVERSITY OF POPULATIONS

To study the dynamics of genetic diversity after connectivity changes, we consider diploid individuals in a finite island model composed of $n$ random mating populations of size $N$, so that the total
population size is \( nN \). The populations exchange migrants at a rate \( m \). The mutations follow the infinite allele model (each mutation produces a new allele; KIMURA and CROW 1964) and occur at a rate \( \mu \). The generations are non-overlapping (Wright-Fisher model; FISHER 1930; WRIGHT 1931).

Genetic diversity, \( H \), is estimated using the identity by descent \( F \) between pairs of alleles, through the relationship provided by NEI and FELDMAN (1972):

\[
H = 1 - F
\]  

(1)

Further, we characterize within-population genetic diversity \( H_s \) and between-population genetic diversity \( H_b \), using within- and between-population genetic identities, respectively. Within-population genetic identity, \( F_s \), corresponds to the probability that two genes randomly chosen from the same population are identical by descent. Between-population genetic identity, \( F_b \), corresponds to the probability that two genes randomly chosen from different populations are identical by descent. Considering that within- and between-population genetic identities \( F_s \) and \( F_b \) at a given time \( t \) are, respectively, \( F_{s,t} \) and \( F_{b,t} \), their values at the next generation (forward in time), respectively, \( F_{s,t+1} \) and \( F_{b,t+1} \), will follow (SMITH 1970; MARUYAMA 1970; LATTER 1973):

\[
\begin{align*}
F_{s,t+1} &= [a(c + (1 - c)F_{s,t}) + (1 - a)F_{b,t}](1 - \mu)^2 \\
F_{b,t+1} &= [b(c + (1 - c)F_{s,t}) + (1 - b)F_{b,t}](1 - \mu)^2
\end{align*}
\]  

(2a)

where the parameters are:

\[
a = (1 - m)^2 + \frac{m^2}{n - 1}
\]  

(2b)

\[
b = \frac{1 - a}{(n - 1)}
\]  

(2c)

\[
c = \frac{1}{2N}
\]  

(2d)
System of equations 2 can be expressed under matrix notation as:

\[ F_{t+1} = AF_t + B \]  
(3a)

Where:

\[
A = (1 - \mu)^2 \begin{pmatrix} a(1 - c) & 1 - a \\ b(1 - c) & 1 - b \end{pmatrix}  
\]  
(3b)

\[
B = (1 - \mu)^2 \begin{pmatrix} ac \\ bc \end{pmatrix}  
\]  
(3c)

\[
F_t = \begin{pmatrix} F_{s,t} \\ F_{b,t} \end{pmatrix}  
\]  
(3d)

Parameters have the following interpretation: \(a\) is the probability that two genes at the same location before migration are still at the same location after migration (either both migrate to the same location or both do not migrate); \(b\) is the probability that two genes that were at the same location before migration migrated to different locations; \(c\) is the probability that two genes within a population are copies of the same gene; \((1 - \mu)^2\) is the probability that neither of the two randomly chosen genes mutated.

PREDICTING THE DYNAMICS OF GENETIC DIVERSITY

To characterize the impact of connectivity changes on genetic diversity, we analyzed the trajectories of within- and between-population genetic diversity from any initial genetic identity state. Using the equation (2), SMITH (1970) and MARUYAMA (1970) showed that genetic identities converge toward an equilibrium value \(F^{eq}\) (value given in Supporting Information File S1). Extending the results obtained by NEI and FELDMAN 1972 for \(n = 2\) populations, we show that the temporal
dynamics of genetic diversity follow (see appendix A for more details):

\[ \mathbf{F}_t = C_1 \lambda_1^t + C_2 \lambda_2^t + \mathbf{F}^{eq} \]  
\( (4a) \)

Where

\[ \mathbf{F}^{eq} = \begin{pmatrix} F_{s}^{eq} \\ F_{b}^{eq} \end{pmatrix} \]  
\( (4b) \)

\( \lambda_1 \) and \( \lambda_2 \) are respectively the largest and smallest eigenvalues of matrix \( \mathbf{A} \), and they follow:

\[
\begin{align*}
\lambda_1 &= \frac{(1 - \mu)^2}{2} [a(1 - c) + 1 - b + \sqrt{(1 - a(1 - c) + b)^2 - 4bc}] \\
\lambda_2 &= \frac{(1 - \mu)^2}{2} [a(1 - c) + 1 - b - \sqrt{(1 - a(1 - c) + b)^2 - 4bc}]
\end{align*}
\]  
\( (5) \)

\( C_1 \) and \( C_2 \) are column vectors of dimension 2 composed of constant values which depend on the parameters of the model (\( m, \mu, n \) and \( N \)) and on the initial genetic identity \( \mathbf{F}_0 \) (appendix A).

In the next section, we provide from eq. 4a and 5 the temporal change of genetic diversity and derive the corresponding time to reach genetic diversity equilibrium after a connectivity change.

TIME TO REACH GENETIC DIVERSITY EQUILIBRIUM

The change of genetic diversity can be decomposed in two main temporal dynamics: a long-term and a short-term dynamics. Indeed, the temporal change of genetic diversity depends on two components: \( |C_1 \lambda_1^t| \) and \( |C_2 \lambda_2^t| \) (eq. 4a). They both follow an exponential decay and their rate of change depends on \( r_1 = ln(\lambda_1) \) and \( r_2 = ln(\lambda_2) \), respectively (appendix A). As \( 1 > \lambda_1 > \lambda_2 > 0 \), \( |C_2 \lambda_2^t| \) decays more rapidly than \( |C_1 \lambda_1^t| \) (see Supporting Information File S1). Thus \( r_1 \) determines the ultimate (or long-term) change of genetic diversity and \( r_2 \) determines the transient (or short-term) change of genetic diversity.

When migration and mutation rates are small (i.e. \( m << 1 \) and \( \mu << 1 \)) and local population
sizes are large (i.e. \( N >> 1 \)), the decay constants \( r_1 \) and \( r_2 \) follow:

\[
\begin{align*}
  r_1 &= -2\mu - \frac{1}{2N_e} \\
  r_2 &= -2\mu - 2m \frac{n}{n-1} - \frac{1}{2N} + \frac{1}{2N_e}
\end{align*}
\]

(6a)

With

\[
N_e = nN \left( 1 + \frac{(n-1)}{nM} \right)
\]

(6b)

where \( M = 4Nm \) is the scaled migration rate and \( N_e \) is the effective population size of the total population (inbreeding, eigenvalue, variance and mutation effective size are equivalent in the finite island model, Whitlock and Barton 1997). As expected from theory (Whitlock and Barton 1997; Wakeley 1999), in the strong migration limit \((M>>1)\), the effective size is equal to the total population size \( nN \), while in the weak migration limit \((M<<1)\), the effective size is higher than the total population size.

We can estimate the durations of the ultimate and transient changes of genetic diversity, denoted \( t_1 \) and \( t_2 \), respectively. Formally, we define \( t_1 \) and \( t_2 \) as the times needed for \( \lambda_1 \) and \( \lambda_2 \) to be reduced to a number \( \alpha \), where \( \alpha \in [0; 1] \):

\[
\begin{align*}
  \lambda_1^{t_1} &= \alpha \\
  \lambda_2^{t_2} &= \alpha
\end{align*}
\]

(7)

Assuming that migration and mutation rates are small and population sizes are large, \( t_1 \) and \( t_2 \) simplify to (appendix A):

\[
\begin{align*}
  t_1 &= \frac{-ln(\alpha)}{2\mu + \frac{1}{2N_e}} \\
  t_2 &= \frac{-ln(\alpha)}{2\mu + 2m \frac{n}{n-1} + \frac{1}{2N} - \frac{1}{2N_e}}
\end{align*}
\]

(8)

The genetic diversity changes as follows (fig. 1): (i) a convergence of duration \( t_2 \) from the initial genetic diversity value to a transient genetic diversity value and then (ii) a convergence of duration \( t_1 \) to the genetic diversity equilibrium \( H^{eq} \). The time to reach genetic diversity equilibrium, \( t_1 \),
depends only on two terms: the mutation rate (term $2\mu$) and the genetic drift at the total population level (term $\frac{1}{2N_e}$). The duration of the transient dynamics, $t_2$, depends on four terms: the mutation rate (term $2\mu$), the migration rate (term $2m$), the genetic drift in each population (term $\frac{1}{2N}$) and the genetic drift at the total population level (term $\frac{1}{2N_e}$). The convergence to the transient and equilibrium values of genetic diversity can occur on separated timescales (i.e. $t_1 >> t_2$) depending on the parameter values. The timescales $t_1$ and $t_2$ can differ from several orders of magnitude.

When $n > 14$, differences are the highest ($t_1 >> t_2$), in the domain where $\mu << \frac{1}{2N}$ and also when $\mu >> \frac{1}{2N}$ and $m > \mu$. When $n \leq 14$, the same conditions apply for $t_1 >> t_2$ except in a restricted domain where $m \simeq \frac{1}{2N}$ (see appendix A). For example, the duration of the transient dynamics is $t_2 \simeq 134$ and the time to reach equilibrium is $t_1 \simeq 1.5 \times 10^5$ (with $\alpha=5\%$), when 10 populations of size 2,500 with a mutation rate of $10^{-6}$ are connected with a migration rate of 0.01.

**DYNAMICS OF GENETIC DIVERSITY AFTER AN ISOLATION EVENT**

We analyzed the dynamics of genetic diversity after an isolation event, starting with a situation where populations are connected and at their equilibrium value, i.e., within- and between-population genetic diversity $H_s$ and $H_b$ are at the expected connection equilibrium values $H_{s,con}^{eq}$ and $H_{b,con}^{eq}$ (see MARUYAMA 1970; SMITH 1970 and Supporting Information File S1).

We observe (fig. 2) that immediately after an isolation event, within-population genetic diversity decreases due to genetic drift to the point where it reaches the mutation-drift equilibrium of an isolated population $H_{s,iso}^{eq}$ (see Supporting Information File S1 and KIMURA and CROW 1964), at a rate determined by $r_2$ (from eq. 6). Meanwhile, between-population genetic diversity slowly increases due to the differentiation of populations induced by mutations (at a rate determined by $r_1$ from eq. 6). Populations ultimately reach complete differentiation (equilibrium value of $H_{b,iso}^{eq} = 1$). We can show from eq. 2 that following an isolation event, within- and between-population diversities change independently and reach their equilibrium value in $t_2$ and $t_1$ generations, respectively (Supporting Information File S2).
The decrease of population genetic diversity (within) can occur quickly relative to population differentiation (between-population genetic diversity, see fig. 1). After an isolation event, within-population genetic diversity ($H_s$) remains above its expected equilibrium $H_{s,iso}^{eq}$, while between-population genetic diversity ($H_b$) remains below its expected equilibrium $H_{b,iso}^{eq}$. However, the timescales of these non-equilibrium periods differ. When populations are isolated ($m=0$), $H_s$ reaches a value close to its equilibrium value in $t_2\approx 1/2\mu + 1/2N$ generations, while $H_b$ reaches a value close to its equilibrium value in $t_1\approx 1/2\mu$ generations (eq. 8). Therefore, when $\theta << 1$, $H_s$ converges much more quickly than $H_b$, and when $\theta >> 1$, both converge in approximately the same amount of time. For example, assuming $\mu = 10^{-5}$ and $N = 1,000$, $H_b$ is significantly lower than the equilibrium value for approximately $t_1\approx 150,000$ generations while $H_s$ is significantly higher than the equilibrium value for approximately $t_2\approx 6,000$ generations (given $\alpha=5\%$).

**DYNAMICS OF GENETIC DIVERSITY AFTER A CONNECTION EVENT**

We analyzed the dynamics of genetic diversity after a connection event, starting with a situation where populations are isolated and at their equilibrium value $H_{s,iso}^{eq}$ and $H_{b,iso}^{eq}$ (see SMITH 1970; MARUYAMA 1970 and Supporting Information File S1). After a connection event (fig. 3), the genetic diversity accumulated in each population during the isolation period is quickly spread to all populations (fig. 3, fast dynamics in light gray, at a rate determined by $r_2$ from eq. 6). Consequently, within-population genetic diversity quickly increases and reaches a high value that is above its expected connected equilibrium value ($H_{s,con}^{eq}$, fig. 3, see Supporting Information File S1). This process creates a peak of within-population genetic diversity $\Delta H_s$ and a transient excess of genetic diversity between populations $\Delta H_b$ (see fig. 3). Then, due to genetic drift, both the within- and the between-population diversities decrease (slow dynamics in dark gray in fig. 3, at a rate determined by $r_1$ from eq. 6), to the point where the diversities reach the expected value of mutation-migration-drift equilibrium ($H_{s,con}^{eq}$ and $H_{b,con}^{eq}$ from eq. 4b; KIMURA and CROW 1964).

Within- and between-population diversities change successively according to two timescales: first, a fast transient dynamics, followed by a slow asymptotic dynamics (separation of timescales
is derived in appendix A and illustrated in fig. 3). Because the transient dynamics can be shorter than the asymptotic dynamics, the excess of genetic diversity ($\Delta H_s$ and $\Delta H_b$) can be maintained for a very long period (from fig. 1, $t_1$ is longer than 10,000 generations).

**PEAK OF GENETIC DIVERSITY GENERATED BY A CONNECTION EVENT**

In this section, we characterize the peak of within-population genetic diversity, $\Delta H_s$, and the excess of between-population genetic diversity, $\Delta H_b$, observed after a connection event as a function of the mutation rate, the genetic drift, the number of populations and the migration rate after connection.

The exact value of the within-population genetic diversity peak is represented in fig. 4. Assuming that migration and mutation rates are small, we can show that good approximations of the values of $\Delta H_s$ and $\Delta H_b$ are (see derivations in appendix B):

$$
\begin{align*}
\Delta H_s &= (1 - (1 - F_{s,iso}^{eq}) \frac{M_n}{n-1}) \cdot F_{b,con}^{eq} \frac{M}{1 + M \frac{n}{n-1} - \frac{2N}{Ne}} 0.05 \frac{t_2}{t_1} \\
\Delta H_b &= (1 - (1 - F_{s,iso}^{eq}) \frac{M_n}{n-1}) \cdot F_{b,con}^{eq} \frac{1 + M - \frac{N}{Ne}}{1 + M \frac{n}{n-1} - \frac{2N}{Ne}} 0.05 \frac{t_2}{t_1}
\end{align*}
$$

(9a)

Where

$$
F_{s,iso}^{eq} = \frac{1}{1 + \theta}
$$

(9b)

is the expected equilibrium identity within an isolated population (Kimura and Crow 1964), and

$$
F_{b,con}^{eq} = \frac{M}{M + (n-1)\theta(1 + \theta + \frac{n}{n-1}M)}
$$

(9c)

is the expected equilibrium identity between connected populations with the scaled migration rate $M$, the number of populations $n$ and the scaled mutation rate $\theta = 4N\mu$ (Maruyama 1970; Smith 1970). These approximations lead to the largest absolute error when a small number of populations ($n = 2$) is combined with weak mutation ($\theta < 1$) and intermediate migration ($M \simeq 5$).

Nevertheless, this error is small (error smaller than 0.025 for $\Delta H_s$ and smaller than 0.08 for $\Delta H_b$), so equation 9 provides a good approximation of $\Delta H_s$ and $\Delta H_b$ for all $n$, $M$ and $\theta$ values (see
appendix B for more details about the validity of approximation 9).

The peak of genetic diversity increases with the difference between the two timescales ($\Delta H_s$ and $\Delta H_b$ increase with $t_1/t_2$) as genetic diversity increases during the transient phase and decreases during the asymptotic phase. Indeed, when those two phases are separated there is no loss of genetic diversity caused by the asymptotic decay during the transient phase (terms $0.05 t_2$ ≃ 1 in eq. 9).

In the domain where the peak is the largest ($M >> 1$ and $\theta << 1$), $\Delta H_s$ and $\Delta H_b$ reach the same value:

$$\Delta H_{\text{max}} = \frac{n - 1}{n} \frac{1}{1 + n\theta}$$

In this domain, $\Delta H_{\text{max}}$ is maximized when the number of populations is (dashed line in fig. 4B):

$$n^* = 1 + 1/\sqrt{\theta}$$

The corresponding peak of genetic diversity, reached at $n^*$, is $\Delta H_{\text{max}}|_{n^*} = \frac{1}{1 + 2\sqrt{\theta}}$. Interestingly, the number of populations and the peak of diversity have a non-monotonous relationship. The peak of genetic diversity decreases when the number of populations approaches 2 and when it tends to infinity, while an intermediate number of populations $n^*$ maximizes the peak of genetic diversity. This can be easily explained by the following processes. During isolation, a small number of populations accumulates less between-population genetic diversity, thus, once reconnected, they share a smaller amount of diversity. In contrast, a large number of populations accumulates a higher level of genetic diversity but also has a higher connection equilibrium value; thus, once reconnected, diversity reaches its expected equilibrium and no peak of diversity is observed.

In summary, high peaks of genetic diversity ($\Delta H_s > 0.25$ in fig. 4) can occur for a large range of the parameter space: when mutation is weak ($\theta < 0.05$) and migration is moderate to strong ($M > 0.5$). Under these conditions, drastic genetic diversity changes can be observed ($\Delta H_s$ values larger than 0.95, fig. 4B for $M \geq 50$ and $\theta < 5 \times 10^{-4}$). The number of populations that maximize the peak of diversity, $n^*$, ranges from a few populations when $\theta \approx 1$, up to a few hundred populations.
when $\theta=10^{-6}$ (values of $\theta<10^{-6}$ are expected to be very rare, they would require a mutation rate of $2.5 \times 10^{-12}/bp$ for a 1 kb gene and a population size of 100). Interestingly, a significant peak of genetic diversity is also observed when only two populations reconnect ($\Delta H_{max}^{n=2}=0.5$ fig. 4B).

PEAK OF GENETIC DIVERSITY RESULTING FROM A MIGRATION RATE INCREASE

Complete isolation of populations is not required to generate peaks of genetic diversity. Indeed, an abrupt increase of migration can generate the peak of genetic diversity characterized in the previous sections. In Supporting Information File S3, we determined that if migration crosses a threshold value $M_T$, peaks of genetic diversity can occur. The value of the threshold $M_T$, assuming that $m \ll 1$ and $\mu \ll 1$, is:

$$M_T = (n-1)\theta \frac{1 + \theta}{1 + n\theta}$$

Consequently, an increase of migration from $M_0$ to $M$ crossing the threshold value $M_T$ (i.e. $M_0 \ll M_T$) generates a peak of genetic diversity that can be approximated by eq. 9 (see Supporting Information File S3). For example, in a subdivided population of $n=10$ and $\theta=0.1$, an increase in migration from $M_0=0.01$ to $M=10$ (which crosses the migration threshold $M_T=0.495$, eq. 12), generates a peak of within-population diversity of 0.350, while a reconnection event in a similar situation would generate a peak of similar intensity (0.358).

IMPLICATIONS FOR THE INFERENCE OF DEMOGRAPHY AND SELECTION

To describe the impact of migration changes on the inference of demography and selection from genetic data, we described the dynamics of two broadly used summary statistics: the Ewens-Watterson statistics (Watterson 1978) and Tajima’s D (Tajima 1989). Both the Ewens-Watterson statistics and Tajima’s D are known to detect an excess (resp. deficit) of rare alleles, which induces negative (resp. positive) values of the statistics, compared with the expected neutral equilibrium (constant size population without selection). Usually, an excess of rare alleles is interpreted either as the signature of balancing selection or population expansion, and a deficit of rare alleles...
is interpreted as the signature of directional selection or as a population bottleneck. We used the Ewens-Watterson statistics, which we denote $H_{EW}$, and follows (Watterson 1978):

$$H_{EW} = H_s - H_A$$

(13)

where $H_s$ is the genetic diversity, and $H_A$ is the expected genetic diversity given the observed number of alleles $K$. We also used Tajima’s D, which we denote $D_T$, and follows (Tajima 1989):

$$D_T = \frac{\pi - S/a_1}{\text{Var}(\pi - S/a_1)}$$

(14)

where $a_1 = \sum_{i=1}^{m} \frac{1}{i}$, $m$ is the sample size, $\pi$ is the average number of pairwise nucleotide differences, and $S$ is the number of segregating sites.

We simulated samples of 50 sequences of 1 kb, with a per nucleotide mutation rate of $2.10^{-8}$, in 4 populations of size 2500, and ran 5,000 replicate simulations. We simulated an isolation event, where the migration rate changed from 0.002 to 0, and a reconnection event in which the migration rate changed from 0 to 0.002. The simulations were performed with the software fastsimcoal (Excoffier and Foll 2011), and the data analysis was performed with Arlequin (Excoffier and Lischer 2010). We simulated samples from the same population (with the parameter values that were used, the sampling scheme had a very weak impact; see Chikhi et al. 2010 for a discussion of how the sampling scheme affects the values of Tajima’s D). To allow for convergence of the coalescent algorithm, we always assumed that the populations were connected prior to the isolation phase. In the reconnection event simulations, we set the duration of the isolated phase to $10N$, which allowed the genetic diversity values to reach their equilibrium value.

We followed the dynamics of the statistics and estimated their distribution as a function of time. The results in figure 5 show that an isolation event produces the same signature on the Ewens-Watterson statistics (fig. 5A) and Tajima’s D (fig. 5B), as expected from a bottleneck event and
from directional selection. Indeed, following an isolation event, genetic drift first causes the elimination of rare alleles and then eliminates more common alleles. Consequently, the number of alleles $K$ decreases more quickly than genetic diversity $H_s$ (fig. 5C). Similarly, the number of segregating sites $S$ decreases faster than the number of pairwise differences $\pi$ (fig. 5D). Therefore, $D_T$ and $H_{EW}$ are skewed toward positive values, as expected after a bottleneck or under the effect of directional selection. Moreover, the statistics remain skewed for a long period of time (less than 10,000 generations in our simulations, see fig. 5).

Results in figure 6 show that a reconnection event can successively produce the same signature as expected from a population expansion or from a bottleneck event on $H_{EW}$ (fig. 6A) and $D_T$ (fig. 6B). Indeed, following a reconnection event, migrants first create an excess of rare variants. The number of alleles $K$ increases more quickly than the genetic diversity $H_s$ (fig. 6C), and the number of segregating sites $S$ increases faster than the number of pairwise differences $\pi$ (fig. 6D), which skews $H_{EW}$ and $D_T$ toward negative values. Second, new alleles brought by migrants increase in frequency, creating an excess of common variants. Consequently $K$ increases more than $H_s$, and $\pi$ increases more than $S$, which skews $D_T$ and $H_{EW}$ toward positive values.

Interestingly, the observed duration of the periods where both statistics are skewed are similar to the expected duration of the dynamics of genetic diversity (from eq. 8). After an isolation event, we observe, in fig. 5, that all statistics reach their equilibrium value within approximately 10,000 generations ($4N$ generations). This duration corresponds to the value of the time required to reach within-population genetic diversity equilibrium after an isolation event, $t_2 \approx 12,000$ generations ($4.8N$ generations, estimated from eq. 8 with $\alpha=5\%$). In this example, genetic drift is stronger than mutation ($2\mu<<1/2N$) and thus $t_2 \sim 2N$. $t_2$ corresponds to the duration of the period where the deficit of rare alleles skews the distribution of $H_{EW}$ and $D_T$. After a reconnection event, we observe fig. 6 that $H_{EW}$ and $D_T$ reach a "peak" within approximately 600 generations ($0.24N$ generations). This duration corresponds to the value of the duration of the transient dy-
namics following a reconnection event, \( t_2 \approx 540 \) generations (0.216\( N \) generations, estimated from eq. 8 with \( \alpha = 5\% \)). In this example, migration is stronger than genetic drift and mutation (\( m >> \mu \) and \( m >> 1/2N \)), and thus, \( t_2 \approx 1/2m \). \( t_2 \) corresponds to the period during which the distribution of \( H_{EW} \) and \( D_T \) is skewed. Subsequently, \( H_{EW} \) and \( D_T \) reach their equilibrium value in approximately 80,000 generations (32\( N \) generations); this duration corresponds to the time required to reach the genetic diversity equilibrium value after a reconnection event, \( t_1 \approx 75,000 \) generations (30\( N \) generations, estimated from eq. 8 with \( \alpha = 5\% \)). \( t_1 \) corresponds to the period during which the deficit of rare alleles is eliminated.

In conclusion, both an isolation and a reconnection event induce changes in the proportion of rare alleles, which skews the values of \( H_{EW} \) and \( D_T \), thus producing a signature that cannot be differentiated from the signature of past demographic events or of selection.

DISCUSSION

We documented a simple neutral mechanism, which creates long-term peaks of genetic diversity. This peak of genetic diversity appears shortly after an abrupt increase in migration and is conserved for a long time. We also demonstrated that such genetic diversity peaks can occur for a large and plausible range of population sizes, migration rates, mutation rates and numbers of populations. Subsequent to the genetic diversity peak, the rate of decay of genetic diversity was slow. Consequently, the mechanisms described here leave a strong and long-term footprint on genetic diversity that affects the Ewens-Watterson statistics and Tajima’s D that are commonly used to infer the history of populations from genetic data.

The peak of genetic diversity is due to the spread of the genetic diversity accumulated during (partial) isolation. Therefore, the migration model that is assumed (island model of migration) is a leading factor in determining the strength of the observed genetic diversity peak. Assuming isolation by distance, the within-population genetic diversity is expected to have locally lower peaks. At the same time, under this assumption, the between-population genetic diversity is expected to
be higher. Indeed, once populations are connected, each population shares with its neighboring populations alleles accumulated during isolation; thus, differentiation between distant populations will be maintained. Additionally, the amount of genetic diversity accumulated during isolation determines the size of the peak of genetic diversity. The maximum value is reached when populations are completely differentiated, i.e., when no alleles are shared between populations. Our results are robust to the relaxation of the complete isolation and complete differentiation assumptions: when isolation is not complete (because of small migration or non-equilibrium genetic diversity), we show that a genetic diversity peak is still observed (see Supporting Information File S3).

A connection event that occurs after an isolation period might play an important role on species diversification. Indeed, we have demonstrated that such events create an excess of genetic diversity. A high level of genetic diversity has often been hypothesized as being a key factor for species diversification. First, evolution from standing genetic variation might be stronger than from de novo mutation (Gibson and Dworkin 2004; Hermisson and Penning 2005; Myles et al. 2005; Barrett and Schluter 2008). Second, both theoretical (Gavrilets 2003; Gavrilets and Losos 2009) and empirical work suggest that a high level of pre-existing genetic diversity in a population increases its rate of diversification (Harmon et al. 2003; Seehauser 2004; Barrett and Schluter 2008). Interestingly, in several cases of adaptive radiation, a high genetic diversity of founder populations has been documented (e.g., Barrier et al. 1999; Bezault et al. 2011). Several authors argued that the connection of populations after a period of isolation might have played an important role in many adaptive radiations (Hughes and Eastwood 2006; Antonelli and Sanmartín 2011; Bezault et al. 2011; Joyce et al. 2011). Therefore, species that experienced population isolation followed by reconnection events could have benefited from a temporary genetic diversity peak which has promoted the diversification of that species. Numerous species are known to have experienced such connectivity changes in the past and show remarkable levels of genetic and species diversity (Arnegard et al. 1999). For example, cichlid fishes in the great African lakes experienced periods of habitat fragmentation and reconnection due to lake wa-
ter level fluctuations (Arnegard et al. 1999); there is some evidence that these processes might have played a role in the explosive radiation of the species (Owen et al. 1990; Young et al. 2009). Additionally, a high rate of speciation is correlated with the timeframe surrounding the uplift of the Northern Andes (Sedano and Burns 2010). The mechanisms described here are thus expected to considerably impact the ability of species to adapt to novel environmental conditions and to diversify over a very long period of time.

Statistics on allelic frequencies such as the Ewens-Watterson statistics (Watterson 1978) and Tajima’s D (Tajima 1989) allow the inference of either selection or population demographic changes. Here, we demonstrate that migration changes can lead to signatures that cannot be differentiated from a selection process or a population size change when using the Ewens-Watterson test and Tajima’s D. Therefore, past migration changes must be considered more carefully and should be viewed as an alternative explanation of bias in neutrality tests and bottleneck or expansion signals. Recently, authors have shown that population structure can bias neutrality tests and produce false bottleneck signals (Leblois et al. 2006; Städler et al. 2009; Chikhi et al. 2010) and that shortly after an isolation event departure from the neutrality can be incorrectly inferred (as shown with simulations by Broquet et al. 2010 and discussed in Waples 2010). The proper interpretation of genetic signatures is crucial for the understanding of the evolutionary history of populations. An interesting extension of this work would be to analyze in more detail the molecular signature of the mechanisms described here and to provide methods that allow the differentiation of such events from selection or demographic changes. Moreover, our results are also relevant for the study of genealogies. Indeed, genetic identities as considered here are commonly used to describe coalescence time distributions (Slatkin 1991; Rousset 1996; Wakeley 1999). Future investigations should also investigate the consequences of isolation and connection events on phylogenetic tree reconstruction. Statistical tools that are available to estimate demographic parameters classically focus on a priori specific scenarios (e.g., population bottleneck, expansion, population with constant migration, population split with subsequent migration; see review in Kuhner 2009).
Given the strong impact of migration changes on genetic diversity, accounting for such scenarios is necessary. Recent methods allowing a larger range of population demographic scenarios, such as Approximate Bayesian Computation (Beaumont et al. 2002; Beaumont 2010), may be powerful tools to disentangle the signature of demographic processes from the observed genetic diversity.

One of the major goals of conservation genetics is to maintain genetic diversity, decrease extinction risks, avoid inbreeding depression, maintain species evolutionary potential and decrease species vulnerability to environmental change (Gilpin and Soule 1986; Newman and Pilson 1997; Jump et al. 2009). In this context, conservationists need to estimate the genetic diversity of a population and its effective size. Such measures are commonly obtained from genetic data and are estimated with standard statistics (Wright 1950; Jorde and Ryman 2007). Although new approaches are emerging that consider populations at a non-equilibrium state, to estimate population size changes and instantaneous migration rates (e.g., Hey and Nielsen 2004), the expected level of genetic diversity is still commonly estimated assuming that populations are at an equilibrium. As shown here, genetic diversity is more likely to be in a transient state. We have demonstrated that reconnecting isolated populations increases genetic diversity above the expected equilibrium value, while isolating populations induces a slow decrease of genetic diversity. Consequently, any estimate inferred from data collected from a population that underwent strong migration changes will not reflect the demographic situation of the population (e.g., census size, genetic diversity). This can have drastic consequences on the selection of conservation strategies and for the management of species (Pearse and Crandall 2004; Caballero et al. 2010).

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Figure 1  The time (in number of generations) $t_1$ to reach genetic diversity equilibrium and the length of the transient dynamics period $t_2$ as a function of the migration rate $m$. The solid line corresponds to $n = 2$ populations, the dashed line to $n = 10$ and the dotted line to $n = 100$. $t_1$ is always at least one order of magnitude higher than $t_2$. This separation of the two periods becomes even greater when $m > \frac{1}{4N} = 10^{-4}$. Parameter values are $N = 2500$, $\mu = 10^{-5}$, $\alpha = 5\%$.

Figure 2  Dynamics of (A) within-population $H_s$ and (B) between-population $H_b$ genetic diversity after an isolation event. Within- and between-population diversity (solid lines) were previously at their respective connection equilibrium $H^{eq}_{s,con}$ and $H^{eq}_{b,con}$. After the isolation event, within- and between-population diversities reach their isolation equilibrium $H^{eq}_{s,iso}$ and $H^{eq}_{b,iso}$ (dashed lines) at rates determined by $r_2$ and $r_1$ (eq. 6). $t_2$ and $t_1$ estimate the time to reach the within- and between-population genetic diversity equilibrium, respectively (eq. 8). Under the effect of genetic drift, within-population diversity reaches its equilibrium value faster than between-population genetic diversity. Parameters are $n = 10$, $N = 2,500$, $m = 10^{-4}$ before isolation and $m = 0$ afterwards, and $\mu = 10^{-5}$. $t_1 \simeq 149,000$ generations and $t_2 \simeq 13,600$ generations (for $\alpha = 5\%$).

Figure 3  Dynamics of (A) within-population genetic diversity $H_s$ and (B) between-population genetic diversity $H_b$ after a reconnection event. Within- and between-population diversities were previously at their respective isolation equilibriums $H^{eq}_{s,iso}$ and $H^{eq}_{b,iso}$ (eq. 4b). After the reconnection event, within- and between-population diversities reach their respective connection equilibriums $H^{eq}_{s,con}$ and $H^{eq}_{b,con}$ (dashed lines). As shown in eq. 8, the time to reach genetic diversity equilibrium $t_1$ and the length of the transient period $t_2$ are well separated. The two periods are: (1) Fast convergence at a rate determined by $r_2$ (eq. 6) that is driven by the spread of diversity that had accumulated within populations during isolation, which creates the peak of within-population diversity ($\Delta H_s$) and the excess of between-population diversity ($\Delta H_b$). (2) Slow dynamics at a rate determined by $r_1$ (eq. 6) that is caused by the gradual loss of genetic diversity. A large number of genera-
tions is needed to reach equilibrium. When \( n = 10, N = 2,500, m = 10^{-4} \) after reconnection, and \( \mu = 10^{-5}, t_1 \simeq 97,000 \) generations, \( t_2 \simeq 6,900 \) generations (for \( \alpha=5\% \)) and \( \Delta H_s \simeq \Delta H_b \simeq 0.11 \).

**Figure 4** Peak of within-population genetic diversity \( \Delta H_s \) generated by a reconnection event. (A) Contour plot of \( \Delta H_s \) as a function of \( \theta \) and \( M \), for \( n = 100 \). We can clearly see the highest peak of diversity in the high \( M \) and low \( \theta \) region. (B) Contour plot of the peak of genetic diversity after a reconnection event as a function of \( \theta \) and \( n \), for \( M >> 1 \) (high \( M \) region identified in (A)). In the high \( M \) region, the within-population diversity peak \( \Delta H_s \) and the between-population diversity excess \( \Delta H_b \) are equal. The dashed line represents the number of populations which maximizes the peak of diversity \( n^* = 1 + 1/\sqrt{\theta} \).

**Figure 5** Effect of an isolation event on Ewens-Watterson and Tajima’s D neutrality tests and on related summary statistics. (A) Ewens-Watterson statistics \( (H_{EW}) \), (B) genetic diversity \( (H_s) \), (C) number of alleles \( (K) \), (D) Tajima’s D \( (D_T) \), (E) number of pairwise differences \( (\pi) \), and (F) number of segregating sites \( (S) \). For each statistics, the black line represents the median of the distribution and the light gray surface represents the 97.5% and 2.5% quantiles of the distribution as a function of the number of generations \( t \) after the isolation event. Gray boxes in (A) and (B) represent the expected distribution of the statistics in an isolated population at equilibrium. Values of \( H_{EW} \) and \( D \) after an isolation event are skewed toward positive values (signature of a bottleneck or directional selection), while there was no change in the size of the population. \( K \) and \( S \) decrease more quickly than \( H_s \) and \( \pi \), because rare alleles are eliminated by genetic drift more quickly than common alleles. Coalescence simulations of a 1 kb locus with a mutation rate of \( 2.10^{-8} \) per bp, where 4 populations of size 2,500 are isolated; 5,000 replicates.
Figure 6  Effect of a reconnection event on Ewens-Watterson and Tajima’s D neutrality tests and on related summary statistics. (A) Ewens-Watterson statistics ($H_{EW}$), (B) genetic diversity ($H_s$), (C) number of alleles ($K$), (D) Tajima’s D ($D_T$), (E) number of pairwise differences ($\pi$), and (F) number of segregating sites ($S$). For each statistics, the black line represents the median of the distribution, and the light gray surface represents the 97.5% and 2.5% quantiles of the distribution, as a function of the number of generations $t$ after the isolation event. Gray boxes in (A) and (B) represent the expected distribution of the statistics in an isolated equilibrium population. Values of $H_{EW}$ and $D$ after a reconnection event are first skewed toward negative values (signature of a population expansion or balancing selection) and then toward positive values (signature of a bottleneck or directional selection), while there was no change in the size of the population. $K$ and $S$ first increase more quickly than $H_s$ and $\pi$ because immigrants bring rare alleles, and then $H_s$ and $\pi$ reach a higher value because immigrant alleles increase in frequency. Finally, alleles are eliminated by genetic drift until the statistics reach their expected equilibrium value when populations are connected. Coalescence simulations of a 1 kb locus with a mutation rate of $2.10^{-8}$ per bp, where 4 populations of size 2,500 isolated during 25,000 generations are reconnected with a migration rate $m = 0.002$; 5,000 replicates.
Figure 1
rate of change determined by $r_2$

rate of change determined by $r_1$

Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
APPENDIX A
DYNAMICS OF GENETIC DIVERSITY

In this appendix, we describe the temporal change of genetic diversity (derivation of eq. 4a, 8 and the separation of the dynamics of genetic diversity into two timescales).

**Temporal change of genetic diversity:** The solution to eq. 3 is:

\[ F_t = A^t(F_0 - F^{eq}) + F^{eq} \]  
(A.1)

Denoting \( P \) the transformation matrix with eigenvector \( U_1 \) (associated with \( \lambda_1 \)) as first column, and eigenvector \( U_2 \) (associated with \( \lambda_2 \)) as second column:

\[ P = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \]  
(A.2)

and denoting

\[ \begin{pmatrix} y_{10} \\ y_{20} \end{pmatrix} = P^{-1}(F_0 - F^{eq}) \]  
(A.3)

Eq. A.1 becomes:

\[ F_t = P \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}^t \begin{pmatrix} y_{10} \\ y_{20} \end{pmatrix} + F^{eq} \]  
(A.4)

Further denoting \( C_1 = y_{10}U_1 \) and \( C_2 = y_{20}U_2 \) leads to eq. 4a.

\( F_t \) changes according to two exponential decay functions, \( \lambda_1^t \) and \( \lambda_2^t \). Their rate of change are

\[ \frac{d\lambda_1}{dt} = ln(\lambda_1) \cdot e^{ln(\lambda_1)t} \] and \[ \frac{d\lambda_2}{dt} = ln(\lambda_2) \cdot e^{ln(\lambda_2)t} \], respectively. Thus \( r_1 = ln(\lambda_1) \) and \( r_2 = ln(\lambda_2) \)
are the decay constants which determine the rate of change of functions $\lambda_1^t$ and $\lambda_2^t$.

Therefore, the eigenvalues of matrix $A$ can be used to compute the rates of change of genetic diversity. As $\lambda_1 > \lambda_2$ and both eigenvalues are less than 1, we have $|r_2| > |r_1|$, and thus $C_2 \lambda_2^t$ tends to 0 faster than $C_1 \lambda_1^t$. $r_2$ determines the transient rate of change of genetic diversity, while $r_1$ determines the asymptotic rate of change of genetic diversity.

We now want to simplify the expression of the rates of change of genetic diversity. To do so, we can rewrite eq. 5 as:

$$
\begin{align*}
\lambda_1 &= \frac{(1 - \mu)^2}{2} \left[ a(1 - c) + 1 - b + (1 - a(1 - c) + b) \sqrt{1 - \frac{4bc}{(1 - a(1 - c) + b)^2}} \right] \\
\lambda_2 &= \frac{(1 - \mu)^2}{2} \left[ a(1 - c) + 1 - b - (1 - a(1 - c) + b) \sqrt{1 - \frac{4bc}{(1 - a(1 - c) + b)^2}} \right]
\end{align*}
$$

With $4bc < (1 - a(1 - c) + b)^2$ (as $m << 1$ and $N >> 1$), and given that $\sqrt{1 - x} = 1 - \frac{1}{2}x + o(x)$, eq. 5 further simplifies to:

$$
\begin{align*}
\lambda_1 &= (1 - \mu)^2 \left( 1 - \frac{bc}{1 - a(1 - c) + b} \right) \\
\lambda_2 &= (1 - \mu)^2 \left( a(1 - c) - b + \frac{bc}{1 - a(1 - c) + b} \right)
\end{align*}
$$

Considering that migration rates and mutation rates are small, we can neglect terms in $m^2$, $\mu^2$, $m/N$, $m\mu$ and $\frac{\mu}{N}$, which leads to:

$$
\begin{align*}
\lambda_1 &= 1 - 2\mu - \frac{1}{2nN(1 + \frac{n-1}{nM})} \\
\lambda_2 &= 1 - 2\mu - 2m \frac{n}{n - 1} - \frac{1}{2N} + \frac{1}{2nN(1 + \frac{n-1}{nM})}
\end{align*}
$$

and thus

$$
\begin{align*}
r_1 &= \ln(1 - 2\mu - \frac{1}{2nN(1 + \frac{n-1}{nM})}) \\
r_2 &= \ln(1 - 2\mu - 2m \frac{n}{n - 1} - \frac{1}{2N} + \frac{1}{2nN(1 + \frac{n-1}{nM})})
\end{align*}
$$
Assuming that migration and mutation rates are small, and that local population sizes are large ($N \gg 1$), eq. A.6 simplifies to eq. 6 as $\ln(1 - x) = -x + o(x)$.

**Respective length of the asymptotic and transient dynamics periods:** We denote $t_1$ and $t_2$ the times needed for $\lambda_1^t$ and $\lambda_2^t$ to be reduced to a value $\alpha$, where $\alpha \in [0; 1]$:

\[
\begin{align*}
&\begin{cases} 
\lambda_1^{t_1} = \alpha \\
\lambda_2^{t_2} = \alpha
\end{cases} \quad \text{(A.7)}
\end{align*}
\]

Which leads to:

\[
\begin{align*}
&\begin{cases} 
  t_1 = \frac{\ln(\alpha)}{r_1} \\
  t_2 = \frac{\ln(\alpha)}{r_2}
\end{cases} \quad \text{(A.8)}
\end{align*}
\]

Assuming small migration and mutation rates and large local population sizes, we can replace the expressions of $r_1$ and $r_2$ from eq. 6 into eq. A.8, and we obtain eq. 8. $F_t$ approximately follows:

\[
F_t \approx \begin{cases} 
  F_{eq} + C_1\lambda_1^t + C_2\lambda_2^t & \text{for } t < t_2 \\
  F_{eq} + C_1\lambda_1^t & \text{for } t_2 < t < t_1 \\
  F_{eq} & \text{for } t_1 < t
\end{cases} \quad \text{(A.9)}
\]

**Timescales separation:** This section presents the conditions for $t_1 \gg t_2$. When $t_1 \gg t_2$,

$\lambda_2^{t_2} \simeq 0$ and $\lambda_1^{t_2} \simeq 1$, thus eq. A.9 simplifies to:

\[
F_t \approx \begin{cases} 
  F_{eq} + C_1 + C_2\lambda_2^t & \text{for } t < t_2 \\
  F_{eq} + C_1\lambda_1^t & \text{for } t_2 < t < t_1 \\
  F_{eq} & \text{for } t_1 < t
\end{cases} \quad \text{(A.10)}
\]

Eq. A.10 decomposes the dynamics of $F_t$ into two timescales: a *transient period* of length $t_2$ and an *asymptotic period* of length $t_1 - t_2 \simeq t_1$. For $t > t_1$, the genetic identity is close to its equilibrium value $F_{eq}$, so $t_1$ can be interpreted as the duration of the disequilibrium period.
Demonstration: Equation A.10 is true if there exists a $t$ such that $\lambda^t_1 \simeq 1$ and $\lambda^t_2 \simeq 0$. For simplicity, we consider that $\lambda^t_1 \simeq 1$ if $\lambda^t_1 \geq 0.95$, and that $\lambda^t_2 \simeq 0$ if $\lambda^t_2 \leq 0.05$. Thus, eq. A.10 is a good approximation if there exists a $t$ such that:

$$\begin{cases} 
\lambda^t_1 \geq 0.95 \\
\lambda^t_2 \leq 0.05
\end{cases} \Rightarrow \begin{cases} 
t \leq \frac{\ln(0.95)}{\ln(\lambda_1)} \\
\lambda^t_2 \leq 0.05
\end{cases} \tag{A.11}$$

Thus, showing that

$$\frac{\ln(0.95)}{\ln(\lambda_1)} \leq 0.05 \tag{A.12}$$

demonstrates that for $t = \frac{\ln(0.95)}{\ln(\lambda_1)}$, we have $\lambda^t_1 \geq 0.95$ and $\lambda^t_2 \leq 0.05$. This provides a sufficient proof of proposition A.11.

We can demonstrate that proposition A.12 depends only on the ratio $\frac{t_1}{t_2} = \frac{\ln(\lambda_2)}{\ln(\lambda_1)}$ (see definitions of $t_1$ and $t_2$ eq. 7). Proposition A.12 leads to:

$$\begin{align*}
\frac{\ln(0.95)}{\ln(\lambda_1)} & \leq 0.05 \\
\Leftrightarrow e^{\frac{\ln(0.95)}{\ln(\lambda_1)}} & \leq 0.05 \\
\Leftrightarrow \ln(0.95) \frac{t_1}{t_2} & \leq \ln(0.05) \\
\Leftrightarrow \frac{t_1}{t_2} & \geq \frac{\ln(0.05)}{\ln(0.95)}
\end{align*}$$

Therefore, the condition $\frac{t_1}{t_2} \geq \frac{\ln(0.05)}{\ln(0.95)}$ is necessary and sufficient to prove propositions A.12, A.11 and the validity of equation A.10. As $\frac{\ln(0.05)}{\ln(0.95)} \simeq 58.4$, equation A.10 is valid when $\frac{t_1}{t_2} > 58.4$. Considering that $m$ and $\mu$ are small, and that $N$ is large, this ratio is approximately equal to (from eq. 8):

$$\frac{t_1}{t_2} \simeq \frac{2\mu + 2m \frac{n}{n-1} + \frac{1}{2N} - \frac{1}{2N_e}}{2\mu + \frac{1}{2N_e}} \tag{A.13}$$

From eq. A.13 we can derive the conditions of the timescales separation of the dynamics of genetic diversity (i.e. the parameter values for which $t_1 >> t_2$). When $n > 14$, differences are the
highest \((t_1 >> t_2)\), in the domain where \(\mu << \frac{1}{2N}\) and also when \(\mu >> \frac{1}{2N}\) and \(m > \mu\). When \(n \leq 14\), the same conditions apply for \(t_1 >> t_2\) except in a restricted domain where \(m \approx \frac{1}{2N}\) or \(m<<\frac{n-1}{2nN}\) are required for \(t_1 >> t_2\), see fig. A.1). Indeed, denoting \(A = \frac{\ln(0.05)}{\ln(0.95)}\), \(t_1 > A\) implies that:

\[
M > \frac{A + 1 - 2n + (A - 1)n\theta + \sqrt{D}}{2}\frac{n^2}{n-1}
\]

or \(M < \frac{A + 1 - 2n + (A - 1)n\theta - \sqrt{D}}{2}\frac{n^2}{n-1}\) \hspace{1cm} (A.14a)

Where

\[
D = (A + 1)(A + 1 - 4n) + 2n\theta(A^2 - 1) + n^2(A - 1)^2\theta^2
\]

(A.14b)

For \(\theta >> 1\), we can neglect terms that do not contain \(\theta\), and conditions A.14 simplify to:

\[
M > (A - 1)\frac{n - 1}{n} \theta
\]

(A.15)

Which simplifies to \(M >> \theta\).

For \(\theta<<1\), terms that contain \(\theta\) can be neglected in eq. A.14, which yields the following conditions:

\[
M > \frac{A + 1 - 2n + \sqrt{(A + 1)(A + 1 - 4n)}}{2}\frac{n^2}{n-1}
\]

or \(M < \frac{A + 1 - 2n - \sqrt{(A + 1)(A + 1 - 4n)}}{2}\frac{n^2}{n-1}\) \hspace{1cm} (A.16)

Therefore, eq. A.10 is not valid when \(M\) is between \(\frac{A + 1 - 2n + \sqrt{(A + 1)(A + 1 - 4n)}}{2}\frac{n^2}{n-1}\) and \(\frac{A + 1 - 2n + \sqrt{(A + 1)(A + 1 - 4n)}}{2}\frac{n^2}{n-1}\).

This domain is centered around \(M = \frac{n-1}{n}\), as \(\frac{A + 1 - 2n + \sqrt{(A + 1)(A + 1 - 4n)}}{2}\frac{n^2}{n-1}\) and \(\frac{A + 1 - 2n + \sqrt{(A + 1)(A + 1 - 4n)}}{2}\frac{n^2}{n-1}\) equalize to \(M = \frac{n-1}{n}\), for \(A=4n-1\). The size of this domain decreases when \(n\) increases (fig. A.1), and eq. A.10 is valid for any value of \(M\) in the domain \(\theta<<1\) when \(n > 14\) (as conditions A.16 are relaxed when \(4n>A+1\), with \(A = \frac{\ln(0.05)}{\ln(0.95)}\)).
**APPENDIX B**

**PEAK OF GENETIC DIVERSITY GENERATED BY A RECONNECTION EVENT**

We derive first the value of the peak of within-population genetic diversity, $\Delta H_s$, and the transient excess of between-population genetic diversity, $\Delta H_b$, generated by a reconnection event. Second, we characterize their dependency on the migration rate, mutation rate, size and number of populations.

**Derivation of $\Delta H$:** We denote the vector of genetic diversity excess at the end of the transient dynamics phase as $\Delta H = \begin{pmatrix} \Delta H_s \\ \Delta H_b \end{pmatrix}$.

The genetic diversity at the end of the transient dynamics phase is approximately $H_{t_2} = 1 - (F_{eq} + C_1 \lambda_1^{t_2})$ (from eq. A.9 when $t \simeq t_2$). Thus $\Delta H$ is approximately:

$$\Delta H = H_{t_2} - H_{eq}$$

$$= 1 - (F_{eq} + C_1 \lambda_1^{t_2}) - (1 - F_{eq})$$

$$= -C_1 \lambda_1^{t_2}$$
The peak of genetic diversity depends on $C_1 = y_{10}U_1$ (from eq. A.2 and A.3). To derive the value of $C_1$, we compute $U_1$ and $y_{10}$. The eigenvector $U_1 = \begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix}$ is associated with $\lambda_1$ and follows:

$$AU_1 = \lambda_1 U_1 \quad (B.2)$$

Eq. B.2 leads to the condition that $u_{21} = -\frac{(a(1-c)-(1-b)-\sqrt{\Delta_{red}})}{2(1-a)}u_{11}$, where $\Delta_{red} = (1 - a(1 - c) + b)^2 - 4bc$. We set $u_{11} = 2(1 - a)$, leading to:

$$U_1 = \begin{pmatrix} 2(1 - a) \\ 1 - a(1 - c) - b + \sqrt{(1 - a(1 - c) + b)^2 - 4bc} \end{pmatrix}$$

Similarly, we can determine the eigenvector associated with $\lambda_2$:

$$U_2 = \begin{pmatrix} 2(1 - a) \\ 1 - a(1 - c) - b - \sqrt{(1 - a(1 - c) + b)^2 - 4bc} \end{pmatrix}$$

Denoting the initial value of genetic identity $F_0 = \begin{pmatrix} F_{s,0} \\ F_{b,0} \end{pmatrix}$ and replacing the expression of $U_1$ and $U_2$ in eq. A.3 leads to:

$$Y_0 = \begin{pmatrix} -\frac{(1-a(1-c)-b-\sqrt{\Delta_{red}})}{4(1-a)\sqrt{\Delta_{red}}} & \frac{1}{2\sqrt{\Delta_{red}}} \\ \frac{1-a(1-c)+\sqrt{\Delta_{red}}}{4(1-a)\sqrt{\Delta_{red}}} & -\frac{1}{2\sqrt{\Delta_{red}}} \end{pmatrix} \begin{pmatrix} F_{s,0} - F_{s,con}^{eq} \\ F_{b,0} - F_{b,con}^{eq} \end{pmatrix}$$

Assuming isolation equilibrium for the initial identity leads to $F_{s,0} = F_{s,iso}^{eq}$ and $F_{b,0} = 0$, thus $\Delta H$ simplifies to:

$$\Delta H = \left[(F_{s,iso}^{eq} - F_{s,con}^{eq}) \frac{1-a(1-c)-b-\sqrt{\Delta_{red}}}{4(1-a)\sqrt{\Delta_{red}}} \right] U_1 \lambda_1^2$$

(B.3)

With $4bc << (1 - a(1 - c) + b)^2$ (as $m << 1$ and $N >> 1$), and given that $\sqrt{1-x} = 1 - \frac{1}{2}x + o(x)$,
\[ \Delta H \] further becomes:

\[ \Delta H = - \frac{1}{2(1-a(1-c)+b-\frac{2bc}{1-a(1-c)+b})} \left[ \frac{F_{s,iso}^\text{eq} - F_{s,con}^\text{eq}}{n-1} \frac{1-a(1-c)+b-c}{1-a(1-c)+b} - F_{b,con}^\text{eq} \right] U_1 \lambda_1^{l_2} \]  

(B.4)

Assuming that migration and mutation rates are always small, we can neglect terms in \( m^2, \mu^2, \frac{1}{N^2}, \frac{m}{N}, \text{and} \frac{m}{N} \) and eq. B.4 simplifies to:

\[ \Delta H = - \left[ \frac{F_{s,iso}^\text{eq} - F_{s,con}^\text{eq}}{n-1} - F_{b,con}^\text{eq} \right] \left( \frac{M}{1+M-\frac{2N}{Ne}} \right) \lambda_1^{l_2} \]  

(B.5)

By replacing the term \( \lambda_1^{l_2} \) (eq. 8 with \( \alpha = 0.05 \)) in eq. B.5, we obtain eq. 9.

Eq. 9 provides a good approximation of the size of the peak of within-population genetic diversity \( \Delta H_s \) and of the transient excess of between-population genetic diversity \( \Delta H_b \) in the entire parameter domain. Fig. B.1(A) and (C), and fig. B.2(A) and (C) represent the exact (solid line) and approximate (dashed line; from eq. 9) values of \( \Delta H_s \) and \( \Delta H_b \), respectively, as a function of \( \theta \) and \( M \), for \( n = 2 \) and \( n = 20 \); we can see that the true and approximate values are very close. Fig. B.1(B) and (D), and fig. B.2(B) and (D), represent the absolute error resulting from the use of eq. 9 as an approximation of \( \Delta H_s \) and \( \Delta H_b \), respectively, instead of its exact value, for \( n = 2 \) and \( n = 20 \). Discrepancies between eq. 9 and the true values of \( \Delta H_s \) and \( \Delta H_b \) can first come from the assumption that \( m << 1, \mu << 1 \) and \( N >> 1 \), and second from the assumption of the existence of \( t_2 \) such that \( |C_2\lambda_1^{l_2}| << |C_1\lambda_1^{l_2}| \).

**Maximum peak of diversity after a connection event:** The peak of genetic diversity increases monotonously with \( M \left( \frac{d(\Delta H)}{dM} > 0 \right) \) for any value of \( m, \mu, n \) and \( N \), and decreases monotonously with \( \theta \left( \frac{d(\Delta H)}{d\theta} < 0 \right) \) for any value of \( m, \mu, n \) and \( N \). The peak of diversity is maximized for intermediate values of \( n \), and reached when:

\[ \frac{d(\Delta H)}{dn} = 0 \]  

(B.6)
Figure B.1 (A), (C) Exact (solid lines) and approximate (dashed lines, from eq. 9) values of the peak of genetic diversity $\Delta H_s$, as a function of the strength of migration ($M$) and mutation ($\theta$). In both (A) and (C), the exact and approximate values of $\Delta H_s$ are very close. (B), (D) Absolute error when using eq. 9 to approximate $\Delta H_s$, as a function of $M$ and $\theta$. The maximum absolute error is reached when $M \simeq 5$ and $\theta < 1$ in both (B) and (D). The error decreases when $M > 5$ or $M << 5$. The absolute error increases when $n$ decreases, but remains weak: (B) the maximum absolute error is 0.025 for $n = 2$, and (D) 0.018 for $n = 20$. Consequently, eq. 9 is a good approximation for the peak of genetic diversity whatever the parameter values of $\theta$, $M$ and $n$ considered.
Figure B.2 (A), (C) Exact (solid lines) and approximate (dashed lines, from eq. 9) values of the transient excess of between-population genetic diversity $\Delta H_b$, as a function of the strength of migration ($M$) and mutation ($\theta$). In both (A) and (C), the exact and approximate values of $\Delta H_b$ are very close. (B), (D) Absolute error when using equation A.9 to approximate $\Delta H_b$, as a function of $M$ and $\theta$. The maximum absolute error is reached when $M \approx 1$ and $\theta < 1$, the absolute error is 0.09 for $n = 2$ (B), and 0.016 for $n = 20$ (D). Eq. 9 is a good approximation for $\Delta H_b$ whatever the parameter values of $\theta$, $M$ and $n$ considered.
If we neglect terms in $\mu^2$, $m^2$, $\frac{1}{N_\theta}$, $\mu m$, $\frac{\mu}{N}$, and $\frac{m}{N}$, and assuming small $\theta$ and high $M$, solving eq. B.6 using equation 10 for the peak of genetic diversity leads to equation 11. When $n = n^*$, eq. 10 leads to a peak of genetic diversity of $\Delta H_{max}^{\text{max}}|_{n^*} = \frac{1}{1+2\sqrt{\theta}}$. 