Inference of homologous recombination in bacteria using whole genome sequences

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ABSTRACT

Bacteria and archaea reproduce clonally, but sporadically import DNA into their chromosomes from other organisms. In many of these events, the imported DNA replaces an homologous segment in the recipient genome. Here we present a new method to reconstruct the history of recombination events that affected a given sample of bacterial genomes. We introduce a mathematical model that represents both the donor and recipient of each DNA import as an ancestor of the genomes in the sample. The model represents a simplification of the previously described coalescent with gene conversion. We implement a Monte Carlo Markov Chain algorithm to perform inference under this model from sequence data alignments, and show that inference is feasible for whole genome alignments through parallelization. Using simulated data, we demonstrate accurate and reliable identification of individual recombination events and global recombination rate parameters. We applied our approach to an alignment of 13 whole genomes from the Bacillus cereus group. We find, as expected from laboratory experiments, that recombination rate is higher between closely related organisms and also that the genome contains several broad regions of elevated levels of recombination. Application of the method to the genomic datasets that are becoming available should reveal the evolutionary history and private lives of populations of bacteria and archaea. The methods described in this article have been implemented in a computer software package, ClonalOrigin, which is freely available from http://code.google.com/p/clonalorigin/.
INTRODUCTION

Bacteria and their distant relatives the archaea make up the majority of cellular living organisms. Short generation times combine with enormous population sizes to create tremendous evolutionary potential. It is currently not feasible to track individual organisms in natural conditions to directly observe their evolution. Instead, genomic deoxyribonucleic acid (DNA) sequencing provides a window onto how bacteria disperse, diversify and adapt because DNA contains information of how organisms are related. In bacteria and archaea, genomic DNA is replicated as part of reproduction by binary fission. Changes in genomic DNA can accumulate because replication is unfaithful or due to DNA damage, but might also be introduced by recombining a segment of foreign DNA into the chromosome. Three mechanisms can lead to the introduction of foreign DNA into a bacterial or archaeal cell: transduction, conjugation, and transformation. The transduction process transfers DNA via phage infection (Canchaya et al., 2003). Conjugation requires two cells to come in contact in order for DNA to be transmitted from one to the other (Chen et al., 2005). Transformation is the uptake of naked DNA from the environment and is regulated by a complex machinery (Claverys et al., 2009). Recombination through these three processes has been found to occur frequently in many groups of bacteria, and to be a driving force in their evolution and adaptation (Didelot and Maiden, 2010).

Recombination in bacteria is analogous to gene-conversion rather than crossing-over in sexually reproducing organisms (McVean et al., 2002), in the sense that the recipient and donor cells make asymmetric contributions to the genetic make-up of the resulting bacterium: typically the donor contributes only a small contiguous segment of DNA whereas the recipient contributes the rest of the genome. For a given set of bacterial isolates, it is thus possible to define its clonal genealogy (Guttman, 1997) irrespective of how frequently
recombination happened, by tracing back in time the ancestry of the isolates following the line of ancestry of the recipient (and not that of the donor) whenever recombination took place. The clonal genealogy is a bifurcating tree where each leaf is an isolate and each internal node represents the most recent common ancestor of the samples below it. Homologous DNA that has been inherited by strict vertical descent evolved according to this clonal tree (this is the so-called clonal frame; Milkman and Bridges 1990). However recombination leads to different parts of the genome having different relationships, each of which can be represented by their own “local tree”. Parts of each local tree may be identical to the clonal tree, reflecting vertical descent of DNA, while other parts of the tree can look entirely different due to recombination events bringing in DNA from a different source. Direct evidence for this phenomenon can be found in Multi-Locus Sequence Typing studies (MLST; Maiden 2006) where the phylogenies reconstructed at the various loci can be very different from one another, for example in Helicobacter (Achtman et al., 1999), Bacillus (Priest et al., 2004) or Salmonella (Falush et al., 2006).

In previous work, we developed a method to infer the clonal genealogy of a group of organisms, while simultaneously identifying for each branch of that genealogy the genomic locations where recombination occurred. The implementation of that method in software is called ClonalFrame (Didelot and Falush, 2007). ClonalFrame has proved useful to identify interesting patterns of recombination in a wide variety of organisms including Campylobacter (Sheppard et al., 2008), Neisseria (Didelot et al., 2009d) and Francisella (Larsson et al., 2009).

In order to perform efficient inference, ClonalFrame does not model the source of specific recombination events (Didelot and Falush, 2007). However, this simplification has two important drawbacks. Firstly ClonalFrame can only identify recombination events that
introduced a number of substitutions higher than expected through mutation alone, and
will miss events that introduce fewer changes (Didelot and Falush, 2007). It does not
explicitly account for other signals of recombination, most importantly homoplasy which
occurs when segregating nucleotides at pairs of sites are not consistent with a single tree
(Maynard Smith and Smith, 1998). The signal of homoplasy could be correctly inter-
preted if the source of recombination events was modelled. Secondly, because ClonalFrame
does not provide any information about the source of the recombination events it identifies,
it can not be used to infer patterns of gene flow between groups of bacteria. One solution
is to postprocess the output of ClonalFrame by giving each recombination event a likely
origin (Didelot et al., 2009a), but this is not as accurate as detecting events and origins
at the same time.

Here we introduce a model similar to ClonalFrame, but where the origin of each recom-
bination event is explicitly modelled as a point on the clonal genealogy. The model can
therefore be described informally as a tree representing the clonal genealogy, with some
additional “recombinant edges” going from one point of the tree to another (Figure 1A)
and affecting a subset of the genome. A recombinant edge “arrives” on the tree at the time
that recombination occurred from an (unsampled) contemporary bacterium. The ancestry
of the unsampled bacterium is followed back in time to its most recent common ancestor
with an isolate in our sample giving its “departure” time. The local tree at any given site
can be traced back by considering only the recombinant edges affecting the site (Figure
1B).

We show how inference can be performed under this new model, and demonstrate that
it outperforms ClonalFrame in detecting recombination events in simulated datasets of
a closed recombining population. We also illustrate the use of our new model on a
dataset containing 13 whole genomes of *Bacillus*. The software we developed to perform inference under our new model is called ClonalOrigin and is freely available from http://code.google.com/p/clonalorigin/.

MODEL AND METHODS

Model

We use a tree to model the clonal genealogy of organisms and consider recombination events as localized changes to this tree affecting a small region of DNA, resulting in differing "local trees" for each site. A conceptual representation of this model is given in Figure 1, and the meaning of the mathematical symbols used in the description below are summarized in Table 1.

The tree $T$ represents the clonal genealogy of the sample of $N$ bacteria under study. We assume a coalescent prior for $T$ (Kingman, 1982), which means that if $t_2, ..., t_N$ denote the length of time during which the sample has $2, ..., N$ ancestors respectively, then the probability of the entire genealogy (ie. the coalescence times plus the tree topology) is given by:

$$P(T) = \prod_{i=2}^{N} \exp \left( -\frac{i}{2} t_i \right)$$

The tree $T$ has total branch length $T = \sum_{i=2}^{N} i t_i$, and along the branches of the clonal genealogy, recombination events occur independently at a constant rate $\rho/2$. Therefore the
distribution of the total number $R$ of recombination events is:

$$R|\mathcal{T}, \rho \sim \text{Poisson} \left( \frac{\rho \mathcal{T}}{2} \right)$$

Each of the $i = 1, \ldots, R$ recombination events is characterized by four variables:

1. an “arrival” point $b_i$ on the clonal genealogy
2. a “departure” point $a_i$ on the clonal genealogy
3. the site $x_i$ where the recombination starts along the observed genetic material
4. the site $y_i$ where the recombination ends along the observed genetic material

The pair $(a_i, b_i)$ can be represented as a recombinant edge linking two points of the clonal genealogy with $b_i$ occurring closer to the observed sequences at the tips than $a_i$ (Figure 1A). Since recombination happens at a constant rate on the clonal genealogy, the arrival points are independent and identically distributed uniformly on the clonal genealogy, i.e.:

$$P(b_i|\mathcal{T}) = \frac{1}{\mathcal{T}}$$

Given an arrival point, the recombinant edge reconnects with the clonal genealogy at rate equal to the number of ancestors in the clonal genealogy, as expected under the coalescent model. $a_i$ is therefore distributed as:
\[ P(a_i|b_i, \mathcal{T}) = \exp(-L(a_i, b_i)) \]  

where \( L(a_i, b_i) \) is the sum of the branch lengths of \( \mathcal{T} \) found between the time of \( a_i \) and that of \( b_i \).

We assume that when recombination occurs, it affects a region which is uniformly distributed along the genome and of length geometrically distributed with mean \( \delta \). Therefore when \( B \) blocks of the genome are under study for a total sequence length of \( L \), the prior for \( x_i \) and \( y_i \) are given by (DIDELOT and FALUSH, 2007):

\[
P(x_i = s|\delta) = \begin{cases} 
\delta/(B\delta + L - B) & \text{if } s \text{ is at the beginning of a block} \\
1/(B\delta + L - B) & \text{otherwise.} 
\end{cases}
\]  

and

\[
P(y_i = s|x_i, \delta) = \begin{cases} 
\delta^{-1}(1-\delta^{-1})^{s-x_i} & \text{if } s \text{ is before the end of the block} \\
(1-\delta^{-1})^{s-x_i+1} & \text{if } s \text{ is the end of the block} 
\end{cases}
\]

Let \( \mathcal{R} \) denote the (unordered) list of all recombination events including all their properties. Combining Equations 2 to 6, we get the complete distribution of \( \mathcal{R} \):

\[
P(\mathcal{R}|\mathcal{T}, \rho, \delta) = P(\mathcal{R}|\rho, \mathcal{T})R! \prod_{i=1}^{R} P(x_i|\delta)P(y_i|x_i, \delta)P(b_i|\mathcal{T})P(a_i|b_i, \mathcal{T})
\]
\[
\exp(-\rho T/2)(\rho/2)^R \prod_{i=1}^{R} P(x_i|\delta)P(y_i|x_i, \delta) \exp(-L(a_i, b_i))
\]  

(7)

On each branch of the clonal genealogy and each recombinant edge mutation events occur at rate \(\theta/2\). For simplicity we assume the model of Jukes and Cantor (1969) where all substitutions are equally likely, but our model can equally be used with other mutational processes (Whelan et al., 2001).

In this framework, time is measured in non-dimensional ‘coalescent units’, with per-generation rates given by \(\theta_g = \theta/2N_e\) for mutation and \(\rho_g = \rho/2N_e\) for recombination, where \(N_e\) is the effective population size (which does not need to be known). It is also useful to define the per-site mutation rate \(\theta_s = \theta/L\) and per-site recombination rate \(\rho_s = \rho/[(\delta-1)B+L]\).

Bayesian Inference

Let \(D\) denote the set of sequences for which we want to perform inference, related by a known clonal genealogy \(T\). We assume for the moment that the values of the parameters \(\theta\), \(\rho\) and \(\delta\) are also known. We want to perform inference on the posterior distribution:

\[
P(R|D, T, \theta, \rho, \delta) \propto P(R|T, \rho, \delta)P(D|T, R, \theta).
\]  

(8)

The first term (the prior) is given by Equation 7. In order to compute the second term (the likelihood), we define the local tree \(T_s\) of each site \(s = 1, ..., L\) as the tree obtained by following the recombinant edges for which \(x_i \leq s \leq y_i\) (cf. Figure 1B) and the clonal genealogy otherwise. The data \(D_s\) observed at site \(s\) depends on the ancestry graph only.
through the local tree $T_s$ and therefore the likelihood can be decomposed as:

$$P(D|T, R, \theta) = \prod_{s=1}^{L} P(D_s|T_s, \theta)$$

(9)

where each of the terms in the product can be computed using the pruning algorithm of Felsenstein (1981). This algorithm provides a natural way of dealing with gaps in the alignment by treating them as missing data.

In order to perform inference, we use a Monte-Carlo Markov Chain (MCMC) with reversible jumps (Green, 1995). Briefly, our update scheme is made of two reversible-jump moves: a “remove” move which proposes to remove an existing recombinant edge chosen uniformly at random, and a “add” move which proposes to add a recombination event with properties proposed according to their priors as defined in Equations 3 to 6. These two moves are accepted according to their Metropolis-Hastings-Green ratio as described in Appendix A. We also use non-transdimensional moves proposing to update the departure point, arrival point, starting site and finishing site of an existing recombinant edge, as described in Appendix A.

**Inference using whole genomes**

The previous section described how to infer the recombination events $R$ from some data $D$, assuming knowledge of $T$, $\theta$, $\rho$ and $\delta$. Direct inference could in principle be done when those quantities are unknown by adding MCMC moves for those parameters, including phylogenetic updates as originally proposed by Yang and Rannala (1997) and by Mau and Newton (1997). However, because we are primarily interested in inference using whole
genomes, such a scheme would be unable to converge because the combined parameter space is extremely large and a parallelization scheme is difficult to implement efficiently. When $\mathcal{T}$, $\theta$, $\rho$ and $\delta$ are known, inference can be greatly simplified by noticing that the recombination events affecting the various alignment blocks $b = 1, ..., B$ are independent. In other words, if $D_b$ denotes the subset of the data corresponding to the block $b$ and $R_b$ denotes the subset of recombination events affecting the block $b$ then we have:

$$P(R|D, \mathcal{T}, \theta, \rho, \delta) = \prod_{b=1}^{B} P(R_b|D_b, \mathcal{T}, \theta, \rho, \delta)$$  

(10)

Thus inference when $\mathcal{T}$, $\theta$, $\rho$ and $\delta$ are known can be done even for a large genomic alignment by parallelization of the inference of the recombination events for each alignment region. Alignment regions are induced by genomic rearrangement processes (Darling et al., 2004) and it is convenient to treat them as independent (given $\mathcal{T}$, $\theta$, $\rho$ and $\delta$) as previously proposed (Didelot and Falush, 2007). Furthermore, when whole genomes are being used, the statistical uncertainty on $\mathcal{T}$, $\theta$, $\rho$ and $\delta$ is likely to be small. We therefore decompose the inference for whole genome alignments into a three step process:

Step 1 : Infer the clonal genealogy $\mathcal{T}$ given the data $D$.

Step 2 : Infer the mutation rate $\theta$, recombination rate $\rho$ and average tract length of recombination $\delta$ given the data $D$ and the clonal genealogy $\mathcal{T}$ inferred in Step 1.

Step 3 : Infer independently for each alignment block $b$ the recombination events $R_b$ affecting $b$ given the data $D_b$, the clonal genealogy $\mathcal{T}$ inferred in the first step, and the parameters $\theta$, $\rho$ and $\delta$ inferred in the second step.
In practice, we perform Step 1 using the ClonalFrame algorithm (Didelot and Falush, 2007). Step 2 is performed by running the inference under our model for each alignment block independently, with $\theta$, $\rho$ and $\delta$ treated as additional parameters (cf. Appendix B for the corresponding MCMC moves). The median value inferred for all blocks is then used as a constant value of $\theta$, $\rho$ and $\delta$ when performing Step 3.

RESULTS

Relationship with the Ancestral Recombination Graph

Although we have described our model independently, it is natural to think about it as a simplification of the Ancestral Recombination Graph (ARG) with gene conversion (Wiuf and Hein, 2000; Didelot et al., 2009c). Our model is in fact equivalent to an ARG model in which non-clonal lines of ancestry are not allowed to either recombine or coalesce with each other. These two simplifications can be justified if we consider that the recombination rate ($\rho$) is relatively low. Non-clonal lines carry little ancestral material (of order $\delta/L$) and therefore have a low effective recombination rate, so that they are unlikely to recombine in the full ARG model into two ancestors with non-empty ancestral material. Furthermore, two non-clonal lines that coalesce in the ARG are unlikely to carry overlapping ancestral material and ignoring such events has been shown to have little effect in the cross-over ARG (McVean and Cardin, 2005; Marjoram and Wall, 2006).

The simplifications in our model relative to the ARG are motivated by our desire to perform inference under the model for very large datasets. Inference under the full ARG process is difficult for datasets of non-trivial size (Stumpf and McVean, 2003), but our simpli-
fication implies that each recombinant edge can be added and removed independently in the MCMC which greatly simplifies inference. Furthermore, the blockwise-independence property of our model in Equation 10 allows inference to be performed independently for each region of an alignment, but this property does not hold for the full ARG model since two recombinant edges affecting different regions may coalesce with each other.

In order to test our approximation, we simulated data under both our model and the ARG using SimMLST (Didelot et al., 2009c). In each simulation we considered two loci, with a per-site mutation rate $\theta_s = 0.01$ and a recombination tract length $\delta = 500$bp. Table 2 shows the average value under both models of the following data statistics often used to study recombination: number of segregating sites $S$, number of unique alleles $H$ (Wall, 2000), measure of linkage between the loci $r^2$ (Hill and Robertson, 1968) and measure of homoplasy between the loci $A$ (Maynard Smith and Smith, 1998). When $\rho = 0$ (first row of Table 2), the two models reduce to a tree model and are formally equivalent. For low values of the recombination rate ($\rho/\theta = 0.5$ or 1) the two models are indistinguishable based on the summary statistics considered. As the recombination rate increases, we find that the approximate model generates systematically higher values of $S$, $H$ and $A$ (Table 2). This explains why when inference is performed under our model using data from the ARG the recombination rate tends to be over-estimated (cf. next section). The measure $r^2$ of the linkage remains the same between the two models even for higher values of the recombination rate (Table 2).
Application to simulated data

We used SimMLST to simulate sequence data under the ARG model for a representative range of parameters. We then applied our algorithm to infer the recombination events and rate $\rho$ given the tree, the mutation rate $\theta$ and the recombination tract length $\delta$. We consider sequences of length 10000bp, which is characteristic of genomic alignment block sizes.

Inference on an ARG with $N = 10$ sequences, $\theta = 300$, $\rho = 50$ and $\delta = 236$bp is considered on Figure 2. There are no instances of confidently inferred but incorrect recombination events in this (typical) example, with false-positive recombination intensity being limited to two types. Firstly, the boundary of recombination region is sometimes imperfectly found (e.g. on branch 1 around 5200bp), and secondly the origin may be incorrect (e.g. parent of sequences 2 and 8, 100bp). In both of these cases the error is “small” in the sense that the prediction is close to the true value. Several kinds of uncertainty are captured; the event itself may be uncertain; the arrival branch may be unclear (e.g. an arrival at sequences 9, 10 or their parent at 5500bp); the recombination may have poorly defined boundaries, or the origin may be poorly determined. In the older part of the ancestry, the inference becomes less certain because the data becomes less informative. In addition to making no false-positive claims about recombination event arrival in this example, ClonalOrigin captures a much larger set of the recombined regions than does ClonalFrame. Many events in the full ARG do not change the tree topology, or contain no mutations, and are therefore undetectable. The inferred recombination rate in the ClonalOrigin model has mean $\rho = 62.5$ (95% confidence interval [45.5, 83.6]), and in the ClonalFrame model has mean $\rho = 25.4$ (95% confidence interval [16.0, 37.6]).
Having established that our algorithm can correctly recover simulated recombination events, we consider how many events we capture as we vary other parameters. In Figure 3 we consider the inferred $\rho$ for a range of ARGs simulated with $N = 20$, $\delta = 236$ and varying $\rho = (25, 50, 75, 100, 150, 200, 250, 300, 400)$ and $\theta = (50, 100, 200, 300, 400, 500)$. We average over 10 ARGs for each set of parameters to reduce variability, which can be very large under the ARG model. ClonalOrigin infers $\rho$ much closer to the true value than does ClonalFrame, which tends to underestimate $\rho$ by roughly a factor of 2 because it misses events that have origins close to the departure point on the tree. ClonalOrigin infers the correct recombination rate for low $\rho$, and overestimates $\rho$ when the mutation rate $\theta$ is high. We conjecture that this happens because the full ARG model allows recombination events to recombine and coalesce, for which ClonalOrigin infers additional events to represent the resulting mosaic of origins. Such mosaic imports to the clonal lineage become more common as recombination rate increases, and are easier to detect as mutation rate increases. Therefore the recombination rate inferred by ClonalOrigin corresponds to the true recombination rate in the limit of small $\rho$ (and large $L$), but should be interpreted in terms of the number of distinct recombined tracts (rather than recombination events) as $\rho$ increases.

**Application to a *Bacillus* genomic dataset**

Bacteria from the *Bacillus cereus* group live predominantly in the soil feeding from dead organic matter, but occasionally infect humans where they can inflict diseases ranging from food poisoning to deadly anthrax (Stenfors Arnesen et al., 2008). MLST has been applied to the *B. cereus* group to investigate its population structure and history (Priest et al., 2004; Sorokin et al., 2006). Three major phylogenetic clades have been
found, which do not agree with species designations (Priest et al., 2004; Sorokin et al., 2006; Didelot et al., 2009a). Analysis of MLST data using ClonalFrame found that recombination occurs at a rate approximately a fifth of that of mutation ($\rho/\theta \approx 0.2$) and results in a greater number of substitutions being introduced ($r/m \approx 1.5$) (Didelot and Falush, 2007; Vos and Didelot, 2009; Didelot et al., 2009a).

Since the sequencing of the first genome of *B. cereus* by Ivanova et al. (2003), several more isolates have been fully sequenced (Ravel et al., 2009; Rasko et al., 2004; Xiong et al., 2009; Han et al., 2006; Challacombe et al., 2007). We collected 13 such genomes (9 from clade 1, 3 from clade 2 and 1 from clade 3) summarized in Table 3, and aligned them using progressiveMauve (Darling et al., 2004, 2010). We found $B = 1218$ blocks of homologous sequence shared between all genomes, with lengths ranging from 502bp to 55619bp, and combined length $L = 3636155$ alignment columns. Cumulative alignments of subsets of those 13 genomes indicated that most of the material shared between them is likely to be part of the core-genome shared by all members of the group (Supplementary Figure 1). We applied GenoPlast (Didelot et al., 2009b) to the material not shared by all genomes and found that the rates of gain and loss of material have been approximately constant during the evolution of the sample, except for a recent acceleration of the rate of gain for the genomes in clade 1 (Supplementary Figure 2).

**Application of the step-by-step methodology** The first step of our analysis was to reconstruct the clonal genealogy of the sample using ClonalFrame (Didelot and Falush, 2007). A unique tree topology was inferred, with little uncertainty in the branch lengths (Supplementary Figure 3). The same topology was also found when using UPGMA, Neighbor-Joining, Maximum-likelihood or Minimum Evolution (Supplementary Figure 4).
ClonalFrame found that many recombination events happened during the evolution of the sample, as shown in Supplementary Figure 3. These were found to happen at rate $\rho/\theta = 0.21$ with an interquartile range (IQR) of $[0.20;0.23]$ relative to mutation, to be of average length $\delta=171\text{bp}$ (IQR $[168;175]$) and to result in $r/m = 2.41$ (IQR $[2.37;2.45]$) more substitutions introduced by recombination than by mutation.

We then applied our new model to infer the recombination events ($R_b$), mutation rate, recombination rate and recombination tract length independently for each block of the alignment. The values inferred for each block are shown in Figure 4. The median value for the recombination tract length $\delta$ was 236bp (IQR $[131;537]$). A few blocks however took extremely low or high values, reflecting the limited information available on $\delta$ when working with a single block. The median value for the per-site mutation rate ($\theta_s/2$) was 0.0219 (IQR $[0.0171;0.0277]$). This was found to be fairly constant throughout the blocks. The median value for the per-site recombination rate ($\rho_s/2$) was 0.0087 (IQR $[0.0047;0.0173]$). Higher rates of recombination were found in three regions of the genome (Figure 4). The median inferred value of $\rho/\theta$ was 0.4051, which is almost twice as high as found by ClonalFrame. This reflects the higher sensitivity of our new model to detect recombination.

Finally we completed the analysis by applying our new model again with values of $\delta$, $\theta_s$ and $\rho_s$ fixed to the median values of the previous paragraph.

Patterns of recombination inferred across the genomes We estimated that $\sim240,000$ recombination events occurred since the 13 genomes shared a common clonal ancestor, but most of these events affected the deep branches of the clonal genealogy, where the statistical uncertainty about each event is very high. Figure 5 shows the numbers of recombination events found by our analysis for any recipient/donor combination of branches, relative to
their expectation under the inferred recombination rate using Equation 7. The main pattern in this figure is that genomes recombine more within clades than between clades. The pattern is particularly visible in clade 1, and exists despite our algorithm having increased power to detect recombination between more divergent sequences. This result may not be surprising considering that recombination in bacteria is sequence identity-dependent in the laboratory (e.g. Majewski 2001). In the Bacillus cereus group, genetic exchanges have previously been found to occur more often within than between clades using MLST data (Didelot et al., 2009a). Figure 5 also contains evidence for a weaker sexual isolation between the two subclades of clade 1 consisting respectively of genomes (1, 2, 4, 10, 11, 12) and (3, 5, 9).

The imports inferred on the branch ancestral to genomes 2 and 11 are atypical for two reasons. Firstly, this branch has imported approximately two times more material from external sources (ie. origins above the MRCA of our sample) than expected under the prior, even though the dependency of recombination to sequence identity should limit the frequency of such imports. Secondly, many imports have been detected coming from genomes 1 and 4. Even though such imports represent within-clade recombination, their increased number relative to the prior seems to go beyond the general increase caused by homology-dependency for the rest of the genomes in this small dataset. This increase is evenly distributed along the genome (Supplementary Figure 5).

Figure 6 shows the number of recombination event boundaries found for each region of the alignment. The number of recombination events is higher than average in four regions at positions 0.8Mbp, 1.8Mbp, 3.2Mbp and 4.9Mbp along the genome of ATCC14579. This result confirms that the variation observed when inferring \( \rho \) in step 2 (cf. Figure 4) was not just caused by a lack of information. Such hotspots of recombination have previously been
described in genomic regions under strong positive selection, for example in *Streptococcus* (Lefebure and Stanhope, 2007; Muzzi et al., 2008) and in *E. coli* (Milkman et al., 2003; Touchon et al., 2009). Here the two peaks at 0.8Mbp and 1.8Mbp correspond to regions of important change in GC content (Ivanova et al., 2003). The peak at 3.2Mbp contains a large number of genes annotated with antibiotic and other drug resistance (Ivanova et al., 2003) which may be under positive selection. The peaks at 0.8Mbp and 4.8Mbp are also located near to *rrn* operons (Ivanova et al., 2003) and a tRNA gene array that harbors the integration site for a *Bacillus* site-specific integrative conjugative element (Grohmann, 2010).

**Recombination events inferred in specific regions** Figure 7 shows the recombination events found in the first 2000bp of the two blocks shown by a blue and a green dot (respectively) on Figure 6. The first region is located right at the beginning of the sequence of genome ATCC14579 which corresponds to the origin of replication, where recombination is not particularly prevalent (Figure 6). This region contains the *dnaA* gene and the beginning of the BC0002 gene (Ivanova et al., 2003) which both play essential roles in DNA replication. One of the clearest recombination events detected in this region is an import into genome 11 from the cluster containing genomes 1, 4 and 12 which spans the entire region shown in Figure 7. In another clear event, genome 10 imported the first ∼1000bp also from the cluster 1,4,12. The ancestor of genomes 2 and 11 imported the second half of the region from an ancestor of clade 1. The first ∼100bp (ie. before the start of gene *dnaA*) may have been imported by any of the three genomes of clade 2 from a member of clade 1. With this single (unclear) exception, there have been no inter-clade events in this region. There are however many branches and genomic locations for which no recombination was found at all, for example on the branches above genomes 1, 3, 4 or 9.
The second region shown in Figure 7 is located in the third hotspot of recombination at 3.2Mbp (Figure 6). This region contains the end of the BC3152 gene which produces an arsenate reductase, the BC3153 gene which produces an arsenic-resistance protein, the BC3154 gene which produces a lactoylglutathione lyase and the beginning of the BC3155 gene which produces an arsenic resistance operon repressor (Ivanova et al., 2003). We found many recombination events in this region, and no single branch of the clonal genealogy remains unaffected (Figure 7). Some of these events represent inter-clade recombination, for example the import of the last ~400bp of genome 12 (which belongs to clade 1) from clade 2, or the import of centered on position 1400bp of genome 6 (which belongs to clade 2) from clade 1. Some events have a very clearly defined origin, for example the import of the first ~600bp of genome 4 from the ancestor of genomes 3 and 9, whereas others present more uncertainty, for example the import at position 100bp on genome 5 which could come from at least 6 branches. This second region contrasts with the first one shown in Figure 7 in several respects: the number of recombination events is higher, their tract lengths are on average shorter, and inter-clade events are more frequent. Since the genes in this second region are involved in resistance to arsenic and its compounds (which are often used as pesticides, herbicides or insecticides), these genes are likely to be under positive selective pressure (Petersen et al., 2007) which often implies a higher rate of recombination (Lefebure and Stanhope, 2007; Orsi et al., 2008; Muzzi et al., 2008; Touchon et al., 2009).
DISCUSSION

Recombination and its consequences have previously been detected and quantified in many different ways. The standard population genetic approach starts with the assumption of a randomly mating population, and to infer the rates of mutation and recombination. Information on recombination comes in particular from the patterns of linkage disequilibrium, which have been used to build detailed maps of recombination rates in humans and other eukaryotes (McVean et al., 2002, 2004; Myers et al., 2005; Winckler et al., 2005). This technique has also been applied to bacteria and archaea (Jolley et al., 2005; Wirth et al., 2007; Tanabe et al., 2009; Touchon et al., 2009).

In bacteria and archaea the standard population genetic framework is problematic because of the absence of a mating pool with defined boundaries or homogeneous rates of exchange (Didelot and Maiden, 2010). Recombination occurs in a clonal context, due to the asymmetry of the contributions of donor and recipient cells. These features of prokaryote reproductive biology have motivated us to develop specialized methods of inference. Building on Didelot and Falush (2007), we discuss a new method, ClonalOrigin, to infer recombination from an alignment of whole bacterial genome sequences. We presented an application to 13 genomes of B. cereus, which revealed interesting variation in recombination rates both across lineages (Figure 5) and across the genome (Figure 6).

The size of genomic datasets means that statistical inference can be computationally challenging. ClonalOrigin is based on a model under which recombination patterns in different alignment blocks can be analyzed independently, facilitating parallelization of most calculations. Inference is decomposed into a three step process which infers successively the clonal genealogy, global parameters and recombination events. For the first step we used
ClonalFrame (Didelot and Falush, 2007) which is not strictly statistically correct since it is based on a different model. There is however typically little ambiguity about the clonal genealogy when working with whole genomes, as shown here by the similarity between the clonal genealogy reconstructed by ClonalFrame and the results of a variety of simpler phylogenetic algorithms (Supplementary Figure 4). Furthermore, small differences in clonal genealogies should not affect the results of the second and third step significantly. For the second step we used the median of the global parameters found by each alignment block when unconditioned, which once again is not strictly correct but likely to be close to the truth given the large amount of genomic sequence considered (> 3Mbp) and the relative stability in their inferred values across blocks (Figure 4). Step 3 introduces no further approximation.

ClonalOrigin follows the standard population genetic approach in estimating values of $\theta$ and $\rho$, which are assumed to be constant across the genome. However, the inferred number and size of events on specific branches of the clonal genealogy may differ substantially from the expectation given by $\theta$ and $\rho$. Our model assumes that recombination events are distributed as if the isolates in the sample were uniformly drawn from a randomly mating population (Equation 4), but the inferred events can follow a very different pattern, which provides considerable information on the diverse biological processes influencing recombination rates. Furthermore, although we do not attempt it here, it is possible in principal to infer the DNA substitutions introduced by each recombination event and hence to study its biological consequences, for example in facilitating the spread of beneficial alleles.

Instead of looking in the output for patterns of deviation from prior expectation, it would be more statistically powerful to account for such possibilities in our model. Since our
model is based on the coalescent (Kingman, 1982), it can easily be extended to account for a number of additional biological processes such as population dynamics (Griffiths and Tavare, 1994) or population structure (Nielsen and Wakeley, 2001; Wilson et al., 2003). Such extensions would introduce new parameters which would appear in both the prior for the clonal genealogy (Equation 1) and the prior for recombinant edges (Equation 7) and would therefore make the decomposition into three inference steps problematic. An interesting alternative would be to leave inference as it is and investigate extensions of the model in a postprocessing step using importance sampling (Meligkotsidou and Fearnhead, 2007).

The model we described assumed a constant rate of mutation and recombination along the genome. This assumption held approximately in our example but may not always be appropriate. The model could therefore be extended, for example using a changepoint process for the mutation and recombination rates along the genome as used for example in LDhat (McVean et al., 2004) or DualBrothers (Minin et al., 2005). This would be computationally feasible under the current three step process by fitting a changepoint model to the rates instead of taking the median for the third step.

A key difference of the model underlying ClonalOrigin in comparison with our previous effort ClonalFrame (Didelot and Falush, 2007) is that the origin of recombination events is explicitly modelled. We showed that this difference makes ClonalOrigin more accurate than ClonalFrame (Figures 2 and 3) when detecting recombination from the Ancestral Recombination Graph model (Wiuf and Hein, 2000; Didelot et al., 2009c). Furthermore, it allows a quantification of the genetic flux between lineages (Figure 5) which would not be possible otherwise. Although ClonalOrigin can still detect external imports, ClonalFrame is a more appropriate model when most DNA imports are from an external source into the
sampled population. In such a scenario, the attempts made by ClonalOrigin at inferring the origin of the imports may be detrimental to the detection of these recombination events as compared to ClonalFrame which makes no such attempt (Didelot and Falush, 2007). A cross between the two models could therefore be envisaged, where some events would have an origin as defined by ClonalOrigin and others would introduce novel polymorphism as in ClonalFrame.

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FIGURE LEGENDS

Figure 1: Illustration of our model for a single region of 300bp and a sample of 4 isolates. Part A shows the full graph of ancestry, with the clonal genealogy shown in bold black and two recombination events shown in red and blue. The red event for example affected the positions 50 to 200 of an ancestor of the first isolate at the point $b_1$, and the donor last shared clonal ancestry with the sample at the point $a_1$. Part B shows the local trees for each site. Points $a_i$ are denoted as “departures” of recombinant edges from the tree and $b_i$ are “arrivals”, with $b_i$ occurring closer to the observed sequences at the tips.

Figure 2: Results on simulated data for a single simulation. The clonal genealogy is shown on the left, and each node is given a color. Each horizontal row on the right represents the arrival of recombination on the branch of the clonal genealogy it is aligned with. For each row, the X axis represents the sequence measured in base pairs and the Y axis represents the probability of recombination on a scale from 0 (where the magenta line is most of the time) to 1 (just below the light gray line). ClonalFrame inference is represented by a thin magenta line. ClonalOrigin inference is shown in solid colors according to their reconstructed origin. Small bars above each row correspond to the true recombined regions in the ARG and are colored according to their origin (or in very light gray to represent absence of recombination). For example, on the branch above genome 9, two real events have occurred, both from an “orange” origin. The first one (around position 900) was fairly short and therefore stayed undetected. The second one (around position 5200) was detected by ClonalFrame with posterior probability close to 100% and by ClonalOrigin with posterior probability around 50% and an origin very likely to be orange but which could also be brown or red.
**Figure 3:** Inferred values of $\rho$ relative to true values for many simulated datasets across various parameter values. Shown are values for ClonalFrame (magenta) and ClonalOrigin (blue). For each of the 6 values of $\theta$ we plot the median (thick line) and inter-quartile range (thin line) of the ratio of inferred $\rho$/true $\rho$, considering the combined results for 10 different instances of the ARG. Lines are labeled by the order they appear at $\rho = 400$. The true value of 1 is shown as a horizontal line for comparison.

**Figure 4:** Scatterplots for all blocks of the Stage 2 analysis of *Bacillus*, showing the inferred values of the log-average tract length ($\log(\delta)$), the mutation rate per site ($\theta_s/2$) and recombination rate per site ($\rho_s/2$). A density plot of the scatterplot is shown using grey shading. The median for all blocks is shown in red.

**Figure 5:** Heatmap for the *Bacillus* Stage 3 analysis showing the number of recombination events inferred relative to its expectation under our prior model given the Stage 2 inferred recombination rate, for each donor/recipient pair of branches. The cells in very light grey are the ones for which the ratio would be meaningless because there are less than 3 observed and expected events.

**Figure 6:** Scatterplot of the Stage 2 analysis of *Bacillus* showing the number of recombination event boundaries per site for each block in the alignment of *Bacillus*. Details of the two blocks shown by a blue and green dot are shown in Figure 7.

**Figure 7:** Results of our Stage 3 analysis for two example regions of the *Bacillus* alignment. The representation is the same as for Figure 2. The two regions are shown by a blue and green dot respectively on Figure 6.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<td>$D$</td>
<td>Aligned sequence data</td>
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<tr>
<td>$N$</td>
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<td>Data contained in block $b$</td>
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<td>$D_s$</td>
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**Symbols used for the clonal genealogy**

- $T$: Clonal genealogy
- $t_i$: Length of time during which there were $i$ ancestors in the clonal genealogy
- $T$: Sum of branch lengths of the clonal genealogy

**Symbols used for the recombination events**

- $R$: Set of recombination events and their properties
- $R$: Number of recombination events
- $R_b$: Recombination events affecting block $b$
- $a_i$: Departure point of the recombination event $i$
- $b_i$: Arrival point of the recombination event $i$
- $x_i$: First site affected by the recombination event $i$
- $y_i$: Last site affected by the recombination event $i$
- $T_s$: Local tree at site $s$

**Symbols used for the remaining parameters**

- $\theta/2$: Rate of mutation on the branches of the clonal genealogy and the recombinant edges
- $\theta_s/2$: Per-site rate of mutation
- $\rho/2$: Rate of recombination on the branches of the clonal genealogy
- $\rho_s/2$: Per-site rate of recombination
- $\delta$: Mean of the geometric distribution modelling the length of recombinant segments

Table 1. Table of symbols
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<thead>
<tr>
<th>$\rho/\theta$</th>
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<th>$H$</th>
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**Table 2.** Comparison with the Ancestral Recombination Graph
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</table>

**Table 3.** Genomes of the *Bacillus cereus* group used in this study
APPENDICES

Appendix A: Details of the MCMC moves

Reversible-jump moves

We use two reversible-jump moves: a “remove” move which proposes removal of an existing recombination event chosen uniformly at random, and a “add ” move which proposes adding a recombination event with properties $a^*, b^*, x^*, y^*$ proposed according to their priors as described in Equations 3 to 6.

These two moves are accepted according to the Metropolis-Hastings-Green ratio:

$$
\alpha = \min \left( 1, \frac{P(D|R')}{P(D|R)} \frac{P(R')}{P(R)} \frac{Q(R' \rightarrow R)}{Q(R \rightarrow R')} \text{Jacobian} \right)
$$

(A1)

where $R$ is the old value of the parameter and $R'$ is the proposed value. The Jacobian is equal to one because no transformation of parameter is being done. The first term of the product is the ratio of likelihoods which is calculated using Equation 9. The second term is the ratio of priors which is calculated using Equation 7. This only leaves to calculate the third term, ie. the ratio of proposal distributions. For the “add” move we have (cf. Equation 7):

$$
Q(R' \rightarrow R) = \frac{1}{R + 1} \quad \text{and} \quad Q(R \rightarrow R') = \frac{P(x^*)P(y^*|x^*)\exp(-L(a^*, b^*))}{T}
$$

(A2)

so that the acceptance ratio is:

$$
\alpha = \min \left( 1, \frac{P(D|R')}{P(D|R)} \rho T \frac{P(R')}{2(R + 1)} \right)
$$

(A3)

For the “remove” move, if we note the recombinant edge proposed to be removed with a * we have:
\[
Q(R' \rightarrow R) = P(x^*) P(y^* | x^*) \exp(-L(a^*, b^*)) \frac{1}{T} \quad \text{and} \quad Q(R \rightarrow R') = \frac{1}{R}
\] (A4)

so that the acceptance ratio is:

\[
\alpha = \min\left(1, \frac{P(D| R') 2R}{P(D|R) \rho T} \right)
\] (A5)

**Update of the start and end points of a recombination event**

We describe the move we used to propose increasing the starting site \(x\) of a given recombination event. Equivalent moves were also used to propose decreasing \(x\) and to propose increasing or decreasing the ending site \(y\). We update \(x\) by adding to it an amount \(d \in [0, D]\) (where \(D\) is a fixed quantity; we used \(D = 10\)). The move can thus be written \(x \rightarrow x + d\). \(d\) is proposed according to its relative posterior probability in \([0, D]\), ie:

\[
Q(x \rightarrow x + d) = \frac{P(D|x + d) P(x + d)}{\sum_{j=0}^{D} P(D|x + j) P(x + j)}
\] (A6)

The terms \(P(x + d)\) follow from Equations 5 and 6. The terms \(P(D|x + d)\) are calculated as follows. If the likelihood at site \(s\) is \(L_s\) with the recombinant edge and \(L'_s\) without it, the likelihood for moving \(x \rightarrow x + d\) is \(P(D|x + d) = P(D|x) \times \prod_{k=1}^{d} \left( \frac{L'_{x+k}}{L_{x+k}} \right)\). We thus calculate \(L_{x+k}\) and \(L'_{x+k}\) for all \(k \in [0, D]\) and all values of the terms \(P(D|x + d)\) follow.

We therefore sample from the proposal distribution \(Q(x \rightarrow x + d)\) and accept with probability:

\[
\alpha = \min\left(1, \frac{\sum_{j=0}^{D} P(D|x + j) P(x + j)}{\sum_{j'=-D}^{0} P(D|x + d - j') P(x + d - j')} \right),
\] (A7)
Update of departing and arrival points of a recombination event

We propose a new value of the departing or arrival point of a recombination event by adding a perturbation $\epsilon \sim \text{Normal}(0, 0.1)$ to its age. If the age is decreased, at each bifurcation crossed one of the two daughter branches is followed so that the probability of choosing a given branch at the new time is $2^n$ where $n$ is the number of bifurcations between the old and the new point. If the age is increased then let $-n$ denote the number of bifurcations of the clonal genealogy between the old and the new point. The Metropolis-Hastings acceptance probability of this move is therefore:

$$
\alpha = \min \left( 1, 2^n \frac{P(D|R')} {P(D|R)} \frac{P(R)} {P(R')} \right)
$$

(A8)

where the ratio of prior can be calculated using Equation 7 and the ratio of likelihoods can be calculated by applying Equation 9 only for the sites affected by the recombination event.

Appendix B: Additional moves for the parameters $\theta$, $\rho$ and $\delta$

Here we described the additional moves of the MCMC required when $\theta$, $\rho$ and $\delta$ are treated as additional parameters as is required in Step 2 of the analysis.

Update of $\theta$

We used an improper Uniform prior on $[0, \infty)$ for the mutation rate $\theta$ and updated its value by proposing the addition of a perturbation $\epsilon$ drawn from Uniform([-5;5]). Since this proposal is symmetric and the prior is uniform, the Metropolis-Hastings acceptance ratio for this move reduces to a ratio of likelihoods which can be computed using Equation 9.
Update of $\rho$

We use a Gamma($\alpha, \beta$) prior for the recombination rate $\rho$. This has the advantage to be conjugate with the distribution of the number of recombination events $R$ given $\rho$ which is Poisson($\rho T/2$). Thus we can deduce that the posterior distribution of $\rho$ is Gamma($R + \alpha, (1/\beta + T/2)^{-1}$). We update $\rho$ by proposing from this distribution, which is a Gibbs move. In the examples shown we used an improper Uniform prior on $[0, \infty)$ for $\rho$ which is obtained by taking $\alpha = 1$ and $\beta = \infty$ and thus the posterior distribution becomes Gamma($R + 1, 2/T$).

Update of $\delta$

Given Equations 5 and 6, the likelihood of the mean length of imports $\delta$ is of the form:

$$L(\delta) = \frac{\delta^X (1 - 1/\delta)^Y}{(b\delta + L - b)^R}$$

(B1)

where $R$ is the number of recombinant edges, $Y = (\sum_{i=1}^{R} y_i - x_i)$ plus the number of import ends falling on block ends, and $X$ is the number of import starts falling on block starts, minus the number of import ends falling before block ends. We assumed an improper Uniform prior on $(0, \infty)$, so that the expression above is the posterior distribution for $\delta$. Update can thus be done by proposing adding a small perturbation $\epsilon$ drawn from Uniform([-5, 5]) and accepting according to the ratio $L(\delta')/L(\delta)$. 

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References


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ORSI, R., Q. SUN and M. WIEDEMANN, 2008 Genome-wide analyses reveal lineage specific contributions of positive selection and recombination to the evolution of Listeria monocytogenes. BMC Evolutionary Biology 8: 233.


A

\[ x_1 = 50 \]
\[ y_1 = 200 \]
\[ x_2 = 1 \]
\[ y_2 = 100 \]

B

From 1 to 50

From 51 to 100

From 101 to 200

From 201 to 300

Figure 1
Figure 3
Figure 4
Figure 6