

Nonparametric Methods for Incorporating Genomic Information Into Genetic Evaluations: An Application to Mortality in Broilers

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ABSTRACT

Four approaches using single-nucleotide polymorphism (SNP) information (F_{∞} -metric model, kernel regression, reproducing kernel Hilbert spaces (RKHS) regression, and a Bayesian regression) were compared with a standard procedure of genetic evaluation (E-BLUP) of sires using mortality rates in broilers as a response variable, working in a Bayesian framework. Late mortality (14–42 days of age) records on 12,167 progeny of 200 sires were precorrected for fixed and random (nongenetic) effects used in the model for genetic evaluation and for the mate effect. The average of the corrected records was computed for each sire. Twenty-four SNPs seemingly associated with late mortality were included in three methods used for genomic assisted evaluations. One thousand SNPs were included in the Bayesian regression, to account for markers along the whole genome. The posterior mean of heritability of mortality was 0.02 in the E-BLUP approach, suggesting that genetic evaluation could be improved if suitable molecular markers were available. Estimates of posterior means and standard deviations of the residual variance were 24.38 (3.88), 29.97 (3.22), 17.07 (3.02), and 20.74 (2.87) for E-BLUP, the linear model on SNPs, RKHS regression, and the Bayesian regression, respectively, suggesting that RKHS accounted for more variance in the data. The two nonparametric methods (kernel and RKHS regression) fitted the data better, having a lower residual sum of squares. Predictive ability, assessed by cross-validation, indicated advantages of the RKHS approach, where accuracy was increased from 25 to 150%, relative to other methods.

LARGE amounts of genomic information have become available in recent years for many species of domestic animals (*e.g.*, HAYES *et al.* 2004; WONG *et al.* 2004). Genomic information can be used to detect polymorphisms contributing to variation in economically important traits, such as disease resistance in farm animals. For instance, some authors have found quantitative trait loci (QTL) or genetic markers associated with diseases in chickens (LIU *et al.* 2001; LAMONT *et al.* 2002). Diseases in broilers often increase mortality in farms and in selection nucleus flocks, elevating costs and reducing profitability. Recently, genetic markers known as single-nucleotide polymorphisms (SNP) have attracted attention, because they are very abundant throughout the genome of all species. For instance, WONG *et al.* (2004) mapped ~2.8 million SNPs in the chicken genome. The chicken polymorphism database (ChickVD) is available at <http://chicken.genomics.org.cn/index.jsp> (WANG *et al.* 2005), which constitutes a valuable inventory

of markers potentially usable as predictors of genetic components underlying disease resistance.

In the context of animal breeding, an interesting application of SNPs is in prediction of performance of progeny groups, *e.g.*, progeny of dairy sires, as an alternative to a standard progeny test. Currently, predictions are based on pedigree indexes (information from ancestors), but without using molecular markers. To the extent that there is an association between genetic markers and progeny performance, genomic information might enhance the accuracy of genetic evaluations. For example, SNP information on candidate sires could be used to make early decisions on which of these should be progeny tested. Hence, it is of interest to study whether genotyping sires for SNPs improves the accuracy of predictions over and above that attained with pedigree indexes.

Some methods have been proposed for dealing with the large amount of genomic information currently available (MEUWISSEN *et al.* 2001; GIANOLA *et al.* 2003; XU 2003). An issue is whether or not all SNPs should be included in a predictive model. For example, excluding irrelevant SNPs led to more accurate classification in an association study (LONG *et al.* 2007). Further, incorpo-

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rating the massive genomic information into genetic evaluations is not trivial from statistical and computational points of view. Many issues need to be considered when using SNPs in the context of traditional methods, *e.g.*, the distributional assumptions made, the fact that the number of SNPs can exceed by far the number of observations in the data, and the difficult problem of estimating and interpreting nonadditive genetic effects. GIANOLA *et al.* (2006) and GIANOLA and VAN KAAM (2008, accompanying article, this issue) proposed nonparametric methods for predicting genomic values. These methods use weaker assumptions than traditional fully parametric models and allow accounting for nonadditive effects without explicit modeling.

The objective of this study was to compare a standard method (a Bayesian counterpart of empirical best linear unbiased prediction) against methods including genomic information, for genetic evaluation of mortality using data from a broiler population. Four methods including genomic information were used: three methods using 24 “informative” SNPs (a widely used linear F_∞ -metric regression and two nonparametric methods, kernel regression and reproducing kernel Hilbert spaces regression) and a Bayesian regression using 1000 SNPs along the genome. Mortality in chickens is a trait with low heritability, thus providing a stringent test of the potential effectiveness of genomic assisted evaluations.

MATERIALS AND METHODS

Phenotypic data and SNP selection: Data consisted of binary mortality records from birds between 14 and 42 days of age, referred to as “late mortality” (LM), from a commercial broiler chicken line in the breeding program of Aviagen Ltd. This trait was scored as 0/1 (alive/dead) and recorded under lower hygiene conditions, to resemble the environment in commercial farms, since in the latter, hygiene level can differ from that in the nucleus. The data set included 12,167 records on the progeny of 200 genotyped sires. Prior to the analyses, the individual bird binary records were adjusted for environmental and mate effects as described in YE *et al.* (2006). The sire-specific means of adjusted records were transformed to a log scale (after shifting the distribution to make all records positive), since their distribution was skewed, and standardized by subtracting their grand mean and dividing by their standard deviation in the log scale.

Pedigree information was tracked six generations back, ending up with 1103 sires in the pedigree file. Sires were genotyped for 5523 SNPs chosen among the 2.8 million SNPs identified in the chicken genome sequencing project (WONG *et al.* 2004). Twenty-four SNPs, selected with the filter and wrapper strategy of LONG *et al.* (2007) applied to the same data, were used in this research for three of the four methods including genomic information. The filter reduces the original thousands of SNPs to a smaller number (*e.g.*, 50), by using an information gain measure. In the wrapper step, a naive Bayesian classifier (using cross-validation prediction accuracy) evaluates each SNP subset’s usefulness, eventually arriving at the 24 SNPs having the highest performance (LONG *et al.* 2007).

Models: Four statistical methods, including genomic information plus a standard genetic evaluation procedure

ignoring markers, were implemented to analyze sires’ transformed adjusted progeny means as a response variable. Let \mathbf{y} (200×1) be the data vector consisting of standardized log-transformed means of adjusted records, to which the following models were fitted.

Model 1—standard genetic evaluation (E-BLUP): A genetic evaluation using the pedigree as sole source of genetic information was implemented using a Bayesian approach. The method is a Bayesian equivalent of empirical best linear unbiased prediction for predicting sires’ transmitting abilities, as described by HENDERSON (1975). The linear model was

$$\mathbf{y} = \mathbf{Z}\mathbf{u} + \mathbf{e},$$

where $\mathbf{u} = \{u_i\}$ is a vector of sire effects, u_i is the effect of sire i in the pedigree ($i = 1, 2, \dots, 1103$), and \mathbf{Z} is an incidence matrix of order 200×1103 linking \mathbf{u} to the observed data. *A priori*, the sire effects were assumed to be distributed as $\mathbf{u} \sim N(\mathbf{0}, \mathbf{A}\sigma_u^2)$, where \mathbf{A} is the additive relationship matrix between sires and σ_u^2 is the variance between sires. The residuals \mathbf{e} were assumed distributed as $N(\mathbf{0}, \mathbf{R} = \mathbf{N}^{-1}\sigma_e^2)$, where $\mathbf{N} = \{n_i\}$ is a diagonal matrix, with its diagonal elements n_i being the number of progeny of sire i . This dispersion structure for \mathbf{e} weights the residuals according to the number of progeny each sire has (SORENSEN and GIANOLA 2002; VARONA *et al.* 2007). Independent scale inverse chi-square prior distributions were assigned to the sire and residual variances as $\sigma_u^2 \sim v_u s_u^2 \chi_{v_u}^{-1}$ and $\sigma_e^2 \sim v_e s_e^2 \chi_{v_e}^{-1}$, respectively, where $v_u = 5$ and $v_e = 3$ were the degrees of freedom, and $s_u^2 = 0.1$ and $s_e^2 = 8.67$ were the corresponding scale parameters. Posterior means of the variance components, as well as of sire effects, were calculated using Gibbs sampling, as described by WANG *et al.* (1993, 1994). Posterior means were used as estimates of sire merit (transmitting ability).

Model 2— F_∞ -metric model (linear regression on SNPs): A model with “fixed” additive effects at each of the SNP loci was fitted following the F_∞ -metric model (linear regression on SNPs) (F_∞ -metric) parameterization described by VAN DER VEEN (1959),

$$y = \sum_{i=1}^q x_{ia} \alpha_i + e,$$

where α_{ia} is the regression of phenotype (y) on the additive effect of SNP i , with $q = 24$ being the number of SNPs fitted, and e is a residual. The regressors x_{ia} relating regression coefficients for SNP i to \mathbf{y} were set up as described by VAN DER VEEN (1959) and ZENG *et al.* (2005). The codes used were

$$x_{ia} = \begin{cases} 1 & \text{for a homozygous SNP (say, AA)} \\ 0 & \text{for a heterozygous SNP (say, Aa)} \\ -1 & \text{for a homozygous SNP (say, aa)} \end{cases}$$

with A being the allele with the highest frequency at the i th locus. Residuals were assumed distributed as in model 1. The estimated genomic value (EGV) of each sire was calculated by summing up the product of the regression coefficient estimates for additivity at each SNP times the code of the respective genotype.

Model 3—kernel regression on SNPs: A nonparametric procedure was used to infer effects of the different SNPs combinations of sires on performance without making strong assumptions. Consider the model

$$y_i = g(\mathbf{x}_i) + e_i$$

(GIANOLA *et al.* 2006), where y_i is the transformed average progeny group mean of sire i , as described earlier, and \mathbf{x}_i is a ($q \times 1$) vector representing the genotype of each sire for the

$q = 24$ SNPs. Here, $g(\mathbf{x}_i)$ is some unknown function of the whole SNP genotype for each sire, representing the expected phenotypic value of sires possessing this 24-dimensional SNPs combination, *i.e.*, the conditional expectation function $E(y_i | \mathbf{x}_i)$. The random vector of residuals, $\mathbf{e} = \{e_i\}$, was assumed distributed independently of \mathbf{x}_i and centered at zero. The conditional expectation function given by

$$g(\mathbf{x}) = \frac{\int y p(\mathbf{x}, y) dy}{p(\mathbf{x})}$$

was inferred using the Nadaraya–Watson estimator (NADARAYA 1964; WATSON 1964). Following SILVERMAN (1986) and GIANOLA *et al.* (2006), the numerator and the denominator in the expression above can be estimated as

$$\int y p(\mathbf{x}, y) dy \approx \frac{1}{nh^q} \sum_{i=1}^n y_i K_h(\mathbf{x} - \mathbf{x}_i)$$

and

$$p(\mathbf{x}) \approx \frac{1}{nh^q} \sum_{i=1}^n K_h(\mathbf{x} - \mathbf{x}_i),$$

respectively, where n is the number of sires with SNP information, $q = 24$ (as before) is the dimension of vector \mathbf{x} , and $K_h(\mathbf{x} - \mathbf{x}_i)$ is a kernel function, with smoothing parameter h , which acts as a measurement of “genomic distance” between two SNP combinations: the observed combination and the focal point. The focal point is any SNP genotype combination at which the function $g(\mathbf{x}_i)$ is evaluated. A Gaussian kernel is often employed (NADARAYA 1964; WATSON 1964), but a trinomial kernel was adopted instead (GIANOLA and VAN KAAM 2008), considering the three possible genotypes for every SNP. The incidence of the three genotypes can be described with two free dummy variates. Let d_{i1} and d_{i2} be the number of disagreements between a focal SNP combination and the observed combination in sire i for two of the three possible genotypes. In this case, the observed genotypes were AA , Aa , or aa , with A being the allele with the largest frequency, and $d_{\bullet 1}$ and $d_{\bullet 2}$ were calculated as the sum of disagreements over loci from observing genotypes at the corresponding locus. An agreement was coded as 0, whereas a disagreement was coded as 1. Table 1 illustrates the values that $d_{\bullet 1}$ and $d_{\bullet 2}$ take at a given locus for each of the nine possible combinations of observed and focal genotypes. The trinomial kernel function had the form

$$K_{h_1, h_2}(\mathbf{x} - \mathbf{x}_i) = h_1^{d_{i1}} h_2^{d_{i2}} (1 - h_1 - h_2)^{2q - d_{i1} - d_{i2}},$$

where \mathbf{x} is the focal genomic combination and \mathbf{x}_i is the observed genomic combination in sire i . The smoothing parameters h_1 and h_2 take values in the interval $(0, 1)$ and also satisfy $0 < h_1 + h_2 < 1$ so that $K_{h_1, h_2}(\mathbf{x} - \mathbf{x}_i)$ is a suitable candidate kernel. The EGV for each sire was the nonparametric estimate using its corresponding genomic configuration as a focal point. The smoothing parameters (h_1, h_2) were tuned using the generalized (direct) cross-validation method described in WAHBA *et al.* (2002). For simplicity, a single parameter $h_1 = h_2 = h$ was tuned.

A Java code application was developed for computing the kernel regression on SNPs, which is available from the authors upon request.

Model 4—reproducing kernel Hilbert spaces regression: A second nonparametric procedure, a reproducing kernel Hilbert spaces (RKHS) regression, was used to estimate the effect of different genomic combinations on LM without making strong assumptions. A brief description of the RKHS is given here, and

TABLE 1

Disagreement scores for the heterozygote (d_{i1}) and homozygote (d_{i2}) dummy variates considered in the trinomial kernel, regarding the focal and the observed genotype

Focal genotype	Observed genotype		
	AA	Aa	aa
AA	$d_{i1} = 0$ $d_{i2} = 0$	$d_{i1} = 1$ $d_{i2} = 0$	$d_{i1} = 0$ $d_{i2} = 1$
Aa	$d_{i1} = 1$ $d_{i2} = 0$	$d_{i1} = 0$ $d_{i2} = 0$	$d_{i1} = 1$ $d_{i2} = 1$
aa	$d_{i1} = 0$ $d_{i2} = 1$	$d_{i1} = 1$ $d_{i2} = 1$	$d_{i1} = 0$ $d_{i2} = 0$

additional details can be found in KIMELDORF and WAHBA (1971), WAHBA (1990, 1999), MALLICK *et al.* (2005) and GIANOLA and VAN KAAM (2008). This method assumes that distances in the Euclidean space can be represented in an isomorphic and isometric space, with a kernel matrix measuring distances between objects (focal points) in the Hilbert space. In our case, the focal points are the SNP genotypes of each individual, and the kernels involve the distance between objects. In these spaces, the penalized sum of squares has the form

$$J[g(\mathbf{x}) | \lambda] = \frac{1}{2} [\mathbf{y} - \mathbf{X}\boldsymbol{\beta} - g(\mathbf{x})]' \mathbf{R}^{-1} [\mathbf{y} - \mathbf{X}\boldsymbol{\beta} - g(\mathbf{x})] + \frac{\lambda}{2} \|g(\mathbf{x})\|_H^2 \quad (1)$$

(WAHBA 1990, 1999), where $\boldsymbol{\beta}$ is a vector of nuisance parameters, \mathbf{X} is an incidence matrix, \mathbf{R} is the residual covariance matrix, and $g(\mathbf{x})$ is a vector of unknown functions of SNP genotypes. The second term in this equation acts as a penalty, adding up some deviance depending on the value of the unknown parameter λ . The term $\|g(\mathbf{x})\|_H^2$ is a norm under a Hilbert space. KIMELDORF and WAHBA (1971) found that the function $g(\mathbf{x})$ that minimizes (1) admits the representation

$$g(\mathbf{x}) = \alpha_0 + \sum_{i=1}^n \alpha_i K(\mathbf{x} - \mathbf{x}_i),$$

where $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \dots, \alpha_n]'$ is a vector of unknown coefficients, n is the number of sires genotyped, and $K(\mathbf{x} - \mathbf{x}_i)$ is a reproducing kernel used as a basis function, possibly depending on some smoothing parameter(s) h . Since data were preadjusted in advance, effects other than the genomic combinations were not included in the model. In our implementation of RKHS, the intercept α_0 was included in the model as the sole element of $\boldsymbol{\beta}$. The following exponential kernel meets the requirements of the RKHS structure,

$$K(\mathbf{x} - \mathbf{x}_i) = \exp[-\text{Score}(\mathbf{x} - \mathbf{x}_i)],$$

where $\text{Score}(\mathbf{x} - \mathbf{x}_i)$ was a score of similarity between two SNPs sequences. The score assigned to each pair of SNP sequences was based on the pairwise sequence alignment (NEEDLEMAN and WUNSCH 1970), with some modifications. The APPENDIX shows the score system in detail. Contrary to the kernels used in GIANOLA *et al.* (2006) and GIANOLA and VAN KAAM (2008), this kernel does not need tuning smoothing parameters inside of the kernel.

Then, a matrix of kernels \mathbf{K} with dimension 200×200 , and with rows in the form $\mathbf{k}'_j = \{K(\mathbf{x}_i - \mathbf{x}_j)\}$, $j = 1, 2, \dots, 200$, was

constructed. The matrix \mathbf{K} is symmetric and positive definite and can be interpreted as a correlation matrix between genomic combinations. The minimizer $g(\mathbf{x})$ of (1) can be expressed in a vectorial manner as

$$g(\mathbf{X}) = \begin{bmatrix} \mathbf{k}'_1 \\ \vdots \\ \mathbf{k}'_j \\ \vdots \\ \mathbf{k}'_n \end{bmatrix} \boldsymbol{\alpha} = \mathbf{K}\boldsymbol{\alpha}.$$

Embedding this expression in (1) and considering that $\boldsymbol{\beta}$ includes only an intercept, the function to be minimized becomes

$$J[\boldsymbol{\mu}, \boldsymbol{\alpha} | \lambda] = \frac{1}{2}[\mathbf{y} - \mathbf{1}\boldsymbol{\mu} - \mathbf{K}\boldsymbol{\alpha}]'\mathbf{R}^{-1}[\mathbf{y} - \mathbf{1}\boldsymbol{\mu} - \mathbf{K}\boldsymbol{\alpha}] + \frac{\lambda}{2}\boldsymbol{\alpha}'\mathbf{K}\boldsymbol{\alpha},$$

where $\boldsymbol{\mu}$ is a scalar common to all observations and $\mathbf{1}$ is a 200×1 vector of ones. After setting the gradients with respect to $\boldsymbol{\mu}$ and $\boldsymbol{\alpha}$ to $\mathbf{0}$ (MALLICK *et al.* 2005; GIANOLA *et al.* 2006; GIANOLA and VAN KAAM 2008), and noting the dependence on parameters λ , the RKHS regression equations can be formulated in matrix form as

$$\begin{bmatrix} \mathbf{1}'\mathbf{R}^{-1}\mathbf{1} & \mathbf{1}'\mathbf{R}^{-1}\mathbf{K} \\ \mathbf{K}'\mathbf{R}^{-1}\mathbf{1} & \mathbf{K}'\mathbf{R}^{-1}\mathbf{K} + \frac{1}{\lambda}\mathbf{K} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\mu}}_\lambda \\ \hat{\boldsymbol{\alpha}}_\lambda \end{bmatrix} = \begin{bmatrix} \mathbf{1}'\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{K}'\mathbf{R}^{-1}\mathbf{y} \end{bmatrix}, \quad (2)$$

where $\mathbf{R} = \mathbf{N}^{-1}\boldsymbol{\sigma}_\epsilon^2$; recall that $\mathbf{N} = \{n_i\}$, where n_i is the number of progeny of sire i . Equivalently, the RKHS approach can be formulated in terms of the random-effects model

$$\mathbf{y} = \mathbf{1}\boldsymbol{\mu} + \mathbf{K}\boldsymbol{\alpha} + \mathbf{e},$$

with the nonparametric coefficients ($\boldsymbol{\alpha}$) and the residuals assumed to be independently distributed as $\boldsymbol{\alpha} | \lambda \sim N(0, \mathbf{K}^{-1}\boldsymbol{\sigma}_\alpha^2)$ and $\mathbf{e} \sim N(0, \mathbf{R})$, with $\boldsymbol{\sigma}_\alpha^2 = \lambda^{-1}$. This model was fitted within a Bayesian framework with λ unknown. Scale inverse chi-square prior distributions were assigned to $\boldsymbol{\sigma}_\alpha^2$ and to the residual variance; *i.e.*, $\boldsymbol{\sigma}_\alpha^2 \sim \nu_\alpha s_\alpha^2 \chi_{\nu_\alpha}^{-1}$ and $\boldsymbol{\sigma}_\epsilon^2 \sim \nu_\epsilon s_\epsilon^2 \chi_{\nu_\epsilon}^{-1}$, where $\nu_\alpha = 4$ and $\nu_\epsilon = 3$ were the degrees of freedom, and $s_\alpha^2 = 0.75$ and $s_\epsilon^2 = 8.67$ were the respective prior scale parameters. System (2) resembles the well-known mixed-model equations in animal breeding (HENDERSON 1975). In fact, additional fixed and random effects (including parametric genetic effects) can be included in the model if necessary (GIANOLA *et al.* 2006). Finally, the EGV of each sire was the nonparametric estimate of its corresponding genomic combination; *i.e.*, $\hat{g}_i(\mathbf{x}_i) = \mathbf{k}'_i \hat{\boldsymbol{\alpha}}$, where $\hat{\boldsymbol{\alpha}}$ is the posterior mean of $\boldsymbol{\alpha}$. A Fortran-90 software was developed to implement the RKHS regression.

Model 5—Bayesian regression: An adapted version of the Bayesian regression proposed by XU (2003) with Gaussian prior distributions assigned to markers (SNPs) was performed using 1000 SNPs randomly chosen from the whole genome. This model allows each SNP marker to have its own variance, producing differential shrinkage of marker effects. The model was

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}.$$

Here, $\mathbf{b} = \{b_i\}$ is 1000×1 , and b_i is a regression coefficient for SNP i ($i = 1, 2, \dots, 1000$) assumed normally distributed *a priori* as $N(0, \boldsymbol{\sigma}_i^2)$, where $\boldsymbol{\sigma}_i^2$ is the unknown variance associated with marker i . The elements of the incidence matrix \mathbf{X} with dimension equal to the number of records ($n = 200$) times the number of markers ($p = 1000$), relating regression coefficients for SNPs to \mathbf{y} , were set up as in model 2 for additivity. The residuals \mathbf{e} were assumed distributed as $N(\mathbf{0}, \mathbf{R})$, with \mathbf{R}

constructed as in previous models. Details of the Markov chain Monte Carlo (MCMC) sampling method are in XU (2003). The EGV of each sire was inferred by summing up the product of the Bayesian estimates (posterior means) of the regression coefficients, times the code of the respective genotype.

In summary, model 1 accounted for polygenic additive effects, and models 2 and 5 considered only additive effects of markers (24 and 1000, respectively). On the other hand, models 3 and 4 are expected to account for all additive and epistatic effects involving the 24 SNPs chosen. Models 1, 2, 4, and 5 were implemented using Bayesian methods via MCMC procedures, specifically the Gibbs sampler. Each of the analyses was based on a chain of 200,000 iterations, with the first 50,000 iterations discarded as burn-in. There were 15,000 samples used for posterior inference, obtained by drawing every 10th iteration from the chain following burn-in.

Model fit: After obtaining predicted transmitting abilities (PTA) (model 1) and EGVs (models 2–5) of all sires, these were matched with the observed mortality rates (adjusted and raw progeny means) in the original data set. These averages were modeled via a local weighted regression using the PTA or the EGV as an explanatory variable, to examine whether the PTA or the EGV bore any relationship with mortality rates. Local weighted regression is a nonparametric approach to fitting curves to data on the basis of smoothing (CLEVELAND 1979). This method approximates the relationship between mortality rates (response variable) and the PTA or EGV estimates (explanatory variables) locally by a smooth curve based on a parametric function, using locally weighted least squares. Weights are assigned such that points close (in the Euclidean distance) to the predictor value of interest receive a higher weight. For convenience, fitting was such that one-fifth of the points in the plot were allowed to influence the smooth at each value. The regressions were computed using the R software (R DEVELOPMENT CORE TEAM 2007). The mean square error (MSE) for each method was calculated as the average of the squares of the differences between the actual average and the local weighted regression estimate at each point.

Predictive ability: The ability of predicting yet to be observed mortality rates was studied by cross-validation for the four methods. Five subsets of data were generated by excluding 20% of the sire progeny means at random. The process was without replacement, so that all sires had missing values in only one of the subsets. This analysis was done for each method, using the variances and smoothing parameters estimated previously. The estimates for sires without phenotypic records were obtained by treating the missing data via the data augmentation algorithm (TANNER and WONG 1987). Thus, each sire had a PTA or an EGV for scenarios with and without records for each of the five methods.

Correlations between predicted and observed adjusted means of the progeny of each sire were calculated for each subset and method. Also, a “global” correlation was calculated using the predicted values from the five subsets together. The model producing the highest correlation was regarded as the one with the best predictive ability of yet to be observed records.

RESULTS AND DISCUSSION

Variance components and parameters: Figure 1 gives a description of the data used in the analyses. Average mortality was 5%. Adjusted mortality rates had a skewed distribution, with a longer tail to the right. However, the

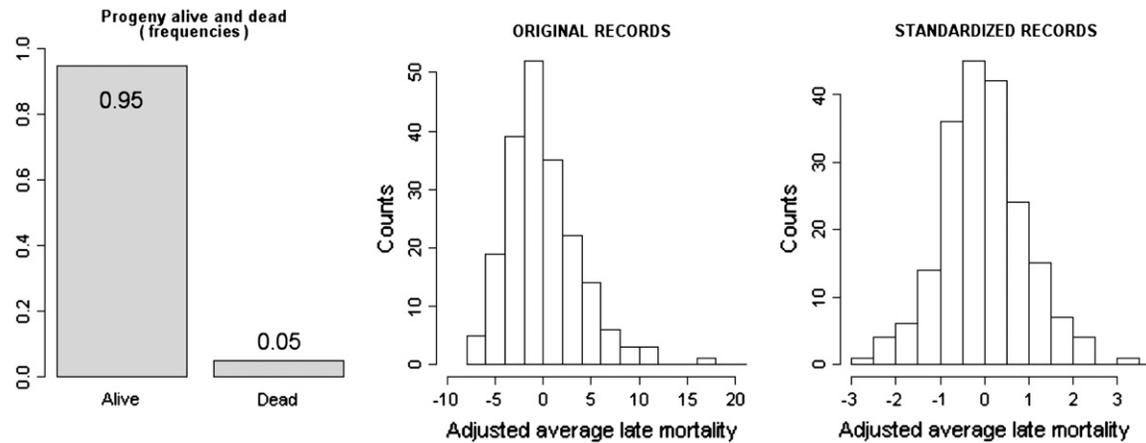


FIGURE 1.—Relative frequencies of live and dead birds (left), distribution of adjusted progeny means of sires (center), and distribution of standardized transformed adjusted progeny means of sires (right). Standardized progeny mean is $(\ln(y + 15) - \mu) / \sigma$, where y is the original adjusted progeny mean, and μ and σ are the mean and the standard deviation, respectively, of the distribution of log-adjusted means.

standardized log-transformed means had a fairly symmetric distribution with mean 0 and standard deviation of 1, as expected. The combinations of 24 SNPs produced 200 unique genotypes, so that each sire could be identified uniquely from its own genomic combination.

Table 2 shows the variance estimates for models 1, 2, 4, and 5. The posterior mean estimates of the residual variance were 24.38, 29.97, 17.07, and 20.74 for models 1, 2, 4 and 5, respectively. This suggests that RKHS accounted for more variability than the parametric models. Using a larger number of markers with Bayesian regression (BR) did not reduce residual variance relative to RKHS, but it did in comparison to E-BLUP, which is supposed to account for all polygenic additive effects. The posterior mean of the sire variance for the E-BLUP model was 0.10, and the genetic variance explained by the markers (sum of the variances of the 1000 SNPs) for BR was 10 times larger (1.05). Hence, BR seemed to account for more genetic variance than E-BLUP. The heritability estimate of LM using E-BLUP was low (0.02), as expected, and it seems unlikely (on the basis of the posterior intervals) that the true value of this parameter exceeds 0.05. A slightly larger heritability (0.06) was reported by DE GREFF *et al.* (2001) for ascites-related mortality, with even higher heritability estimates for heart-failure syndrome-related mortality (0.10), major heart-lung-related mortality (0.15), and total mortality (0.22). JANSSEN and BOLDER (2000) found a heritability estimate of 0.12 for mortality 28 days after infection with *Salmonella*. However, comparisons should be done cautiously, because traits were different and the estimates in the present study were based on 200 sires only (*e.g.*, note that the 95% highest posterior density regions for heritability ranged from 0.004 to 0.05). Also, estimates with linear models are frequency dependent. The nonparametric coefficient variance (σ_{α}^2 , the reciprocal of the smoothing parameter λ) was estimated at 0.40

(0.07) with RKHS. The interpretation of this parameter is not obvious; in general, the smaller it is, the smoother $\hat{g}(\mathbf{x})$ is (HASTIE and TIBSHIRANI 1990). Model 3 (kernel regression) does not lead to variance estimates directly. The h smoothing parameter for kernel regression was estimated at 0.45. Comparison of these nonparametric results with those from other studies is not possible, since such methodology has not been used previously in quantitative genetics. The choice and design of kernels are of great importance in nonparametric regression, and they deserve further research for applications in genomic selection.

Figures 2 and 3 show the nonparametric fits relating adjusted and raw mortality in the progeny to the PTA or the EGV, respectively, for each of the five models. There was an association between PTA or EGV and progeny mortality. Sires with higher PTA or EGV had higher progeny mortality rates. Kernel regression and RKHS appeared to reproduce the data accurately, with a much smaller dispersion of average progeny LM across the regression surface than other methods. The kernel regression model fitted almost perfectly the adjusted LM (MSE = 0.70; Figure 2) and produced also the smallest MSE (3.20×10^{-4} ; Figure 3) with raw average progeny mortality. This is probably due to the fact that each sire had a unique marker genotype. The ranked fitted values were in exactly the same order as the ranked sire means. The nonparametric methods produced greater agreement between sire estimates and average mortality of the corresponding progeny group, and the F_{∞} -metric regression model fitted to the data the worst.

Predictive ability: Spearman (r_s) and Pearson (r_p) correlations between estimates of sire effects from the different models ranged from 0.27 to 0.93 (Table 3). E-BLUP had a higher agreement with the BR method ($r_s = 0.91$, $r_p = 0.92$) than with either kernel regression or RKHS. This illustrates that the polygenic model pro-

TABLE 2

Posterior means (*m*), standard deviation (SD) (in parentheses), and highest posterior densities (HPD) regions for the residual (σ_c^2), sire (σ_u^2), nonparametric coefficient (σ_α^2), and marker (σ_m^2) variances, and heritability (h^2), by model

Parameter	Posterior features	E-BLUP	F_∞ -metric	RKHS	BR
σ_c^2	<i>m</i> (SD)	24.38 (3.88)	29.97 (3.22)	17.07 (3.02)	20.74 (2.87)
	HPD (95%)	16.88–32.04	24.33–36.86	11.78–23.64	15.65–26.98
Variance ^a	<i>m</i> (SD)	0.10 (0.06)	—	0.40 (0.07)	1.05 (0.88)
	HPD (95%)	0.03–0.24	—	0.28–0.55	0.67–2.02
h^2	<i>m</i> (SD)	0.02 (0.01)	—	—	—
	HPD (95%)	0.004–0.050	—	—	—

E-BLUP, Bayesian linear model; F_∞ -metric, linear regression on SNPs based on the F_∞ -metric; RKHS, reproducing kernel Hilbert spaces regression; BR, Bayesian regression.

^aSire variance (σ_u^2) for E-BLUP, nonparametric coefficient variances (σ_α^2) for RKHS, and genetic variance associated with the 1000 markers (σ_m^2) for BR.

vides a close approximation to a BR model with a large number of markers having additive effects. The F_∞ -metric genomic evaluation had the weakest correlations with the other methods, including the two nonparametric procedures, even though F_∞ , kernel regression, and RKHS used the same information (phenotype and genomic information). This indicates that the F_∞ approach produced predictions that were distinct from those from other methods. Also, E-BLUP (which does not incorporate genomic information) and the RKHS methods gave different results, and E-BLUP had a higher correlation with BR than with RKHS, as noted earlier.

Results from the predictive cross-validation are shown in Table 4, for each of the methods. RKHS showed better predictive ability in three of the five subsets, whereas the E-BLUP and BR models were better only in one of the subsets each. RKHS had the best global predictive ability when all subsets were pooled, with the correlation being 25, 100, and 150% larger than for BR, E-BLUP, and F_∞ -metric, respectively. Even though LM has a very low heritability, the RKHS incorporating

genomic information from only 24 SNPs attained better predictive ability than either E-BLUP or BR, the latter having 1000 markers in the explanatory structure. The models including genomic information had better predictive ability than E-BLUP in four of five subsets ($P < 0.01$ under binomial sampling), and the RKHS method had a better predictive ability in three of these four subsets ($P = 0.05$ under binomial sampling). Within methods considering genomic information, RKHS and kernel regression models can potentially account for nonadditive genetic effects (*e.g.*, dominance and epistasis). The Bayesian regression did not perform better than RKHS, in spite of using a much larger amount of genomic information. This might be due to the strong assumptions placed on BR such as linearity, multivariate normality, or absence of interactions among SNPs. Further studies should deal with the inclusion of large amounts of information in the RKHS regression model and with a comprehensive comparison between methods incorporating SNPs along the whole genome and those based on filtering SNP information, as in Long *et al.* (2007).

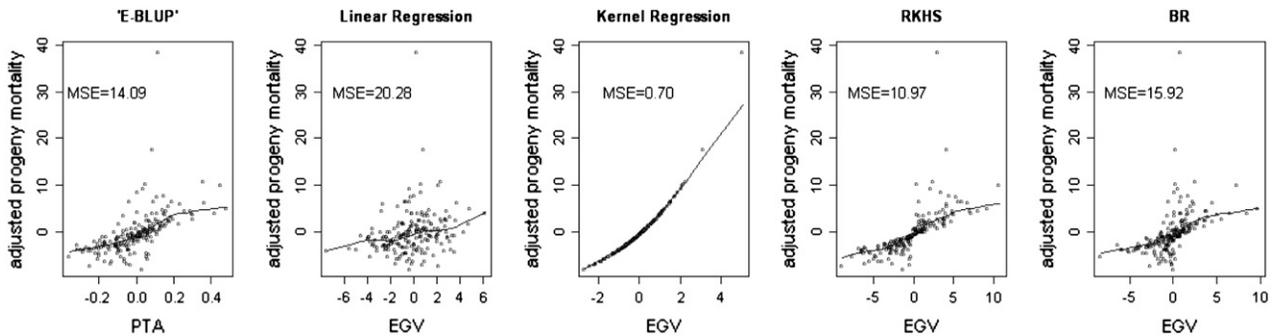


FIGURE 2.—Nonparametric locally weighted regression of adjusted progeny mortality scores on estimated sire predicted transmitting ability (PTA) or genomic value (EGV) for each of the five models. Mean squared errors (MSE) are given for each method. Points in the plots are the adjusted progeny average mortality for each sire. Each method led to different associations between the points and the PTA or the EGV. E-BLUP, Bayesian linear model without genomic information; F_∞ -metric, linear regression on SNPs based on the F_∞ -metric model; kernel regression, nonparametric kernel regression with SNPs within sire treated as a genomic combination; RKHS, reproducing kernel Hilbert spaces regression with SNPs within sire treated as a genomic combination; BR, Bayesian regression on 1000 SNPs based on Xu (2003).

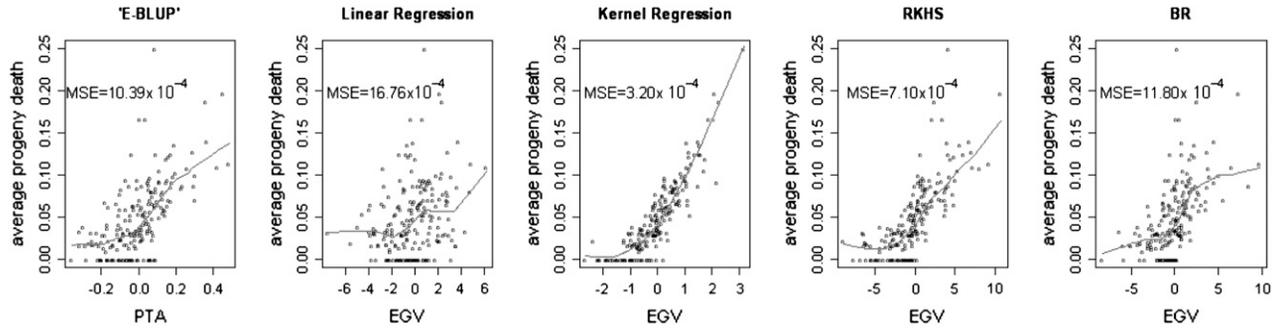


FIGURE 3.—Nonparametric locally weighted regression of raw progeny mortality rate on estimated sire predicted transmitting ability (PTA) or genomic value (EGV) for each of the five models. Mean squared errors (MSE) are given for each method. Points in the plots are the raw progeny average mortality rate for each sire. Each method led to different associations between the points and the PTA or the EGV. E-BLUP, Bayesian linear model without genomic information; F_{∞} -metric, linear regression on SNPs based on the F_{∞} -metric model; kernel regression, nonparametric kernel regression with SNPs within sire treated as a genomic combination; RKHS, reproducing kernel Hilbert spaces regression with SNPs within sire treated as a genomic combination; BR, Bayesian regression on 1000 SNPs based on Xu (2003).

An important difference between the two nonparametric methods was the kernel used. Almost identical results were found when the same kernel was used in kernel regression and in RKHS; however, a trinomial kernel in RKHS would violate theoretical assumptions. Kernel regression is computationally straightforward and fast, because it can be executed without MCMC sampling. However, if some nuisance effects are to be fitted simultaneously with the marker information, a two-step method would be necessary, as described in GIANOLA *et al.* (2006), increasing computational time and software complexity. On the other hand, RKHS is methodologically and computationally self-contained.

Nonparametric methods are expected to have a better predictive ability than parametric models when relationships between variables are cryptic. However, the interpretation of parameters is not straightforward, or even useful, since the focus is on description and prediction. A low heritability and the categorical nature of LM provided a challenge to all methods, but there was some advantage for the RKHS approach using genomic information. Bootstrapping might give a clearer picture of the uncertainty about differences in predictive ability between methods. However, it is computationally demanding.

LEGARRA *et al.* (2007) found that methods incorporating genomic information had better predictive ability than BLUP in a model with independent families. In a simulation, GIANOLA *et al.* (2006) found that RKHS regression had higher accuracy of prediction of genotypic values than linear random regressions, mainly under additive \times additive epistasis. Also, these authors assumed a different penalty structure for the nonparametric coefficients and used a Gaussian kernel. The kernel and scoring system developed in this study performed better than the kernel described in GIANOLA *et al.* (2006) (results not shown).

Other authors have proposed parametric methods for including genomic information into genetic evalua-

tions. For instance, MEUWISSEN *et al.* (2001) compared least-squares, BLUP, and two Bayesian methods with different prior distributions for the QTL effects, using simulated data. They found that Bayesian methods assigning specific prior distributions to marker effects were more reliable, but their priors matched the simulated parameter values. Also, GIANOLA *et al.* (2003) and Xu (2003) dealt with markers over the entire genome to estimate polygenic effects by applying Bayesian methods. Strong assumptions are needed for these methods, as mentioned earlier.

Pedigree and genomic information were not combined in any of the methods used in this study, due to the small number of sires. However, including both sources of information in analyses with a larger amount of data might improve predictive ability of the methods presented here.

Implications: Three methods using information from 24 SNPs associated with chicken mortality in broilers,

TABLE 3

Spearman (above diagonal) and Pearson correlations (below diagonal) between estimates of sire effects from the five different methods

	E-BLUP	F_{∞} -metric	Kernel	RKHS	BR
E-BLUP	—	0.44	0.77	0.84	0.91
F_{∞} -metric	0.48	—	0.33	0.36	0.46
Kernel	0.66	0.27	—	0.93	0.76
RKHS	0.84	0.36	0.79	—	0.85
BR	0.92	0.50	0.58	0.80	—

Standard errors <0.0001. E-BLUP, Bayesian linear model without genomic information; F_{∞} -metric, linear regression on SNPs based on the F_{∞} -metric model; kernel, nonparametric kernel regression with SNPs within sire treated as a genomic combination; RKHS, reproducing kernel Hilbert spaces regression with SNPs within sire treated as a genomic combination; BR, Bayesian regression on 1000 SNPs based on Xu (2003).

TABLE 4

Pearson correlations between predicted and actual values of the progeny average of each sire for late mortality in each subset (20% sires predicted in each subset) and by method

Subset	E-BLUP	F_{∞} -metric	Kernel	RKHS	BR
First	0.03	0.26	0.05	<i>0.27^a</i>	0.13
Second	0.18	0.25	0.28	<i>0.37</i>	0.12
Third	<i>0.18</i>	-0.13	0.06	-0.01	0.17
Fourth	-0.04	0.12	0.13	<i>0.28</i>	0.15
Fifth	0.17	0.06	0.23	0.15	<i>0.25</i>
Global	0.10	0.08	0.14	<i>0.20</i>	0.16

E-BLUP, Bayesian linear model without genomic information; F_{∞} -metric, linear regression on SNPs based on the F_{∞} -metric model; kernel, nonparametric kernel regression with SNPs within sire treated as a genomic combination; RKHS, reproducing kernel Hilbert spaces regression with SNPs within sire treated as a genomic combination; BR, Bayesian regression on 1000 SNPs based on Xu (2003).

^aHigher values indicate more accurate predictions. The highest correlation for each set is in italics.

which is a lowly heritable trait, and one method using 1000 SNPs across the whole genome were used for genomic evaluation of late mortality. Comparison of these methods against a standard genetic evaluation using pedigree information indicated that predictive ability was similar for E-BLUP, LM, kernel regression, and BR, but RKHS increased predictive correlations by 0.04 over BR, by 0.10 over E-BLUP, and by 0.12 over a linear regression on SNPs using the F_{∞} -metric model. The kernel regression and the RKHS procedure fitted the data better than E-BLUP, BR, or F_{∞} -metric, producing a smaller MSE. It seems that incorporating information on SNPs into genetic evaluation is feasible with nonparametric models, with the RKHS being appealing, because of its ability to deal with complex gene action as well as include parametric components in the model (GIANOLA and VAN KAAM 2008). Further, RKHS had the highest accuracy when predicting performance of sires without progeny records in the analysis. This study suggests that RKHS using just some SNPs having an association with the trait may provide better predictive ability than a BR using markers over the entire genome. This is an important issue in the development of genomic selection procedures that must be considered in future studies in a thorough manner.

Nonparametric methods for incorporating genomic information into genetic evaluations should be studied further, since they are appealing for handling large numbers of SNPs and their interactions. At present, SNP chip developments allow genotyping thousands of SNPs for many individuals. If these methods prove to be better than existing ones, breeding companies could genotype candidates and make earlier decisions on breeding programs and progeny testing, with the potential for increasing the rate of genetic progress (SCHAEFFER 2006). Developing suitable kernels to measure the extent by

which haplotypes differ from each other and that allow for weighting the most relevant SNPs differentially constitutes an important research challenge.

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APPENDIX

A score system was developed to account for differences between two sequences of SNPs observed in two individuals (or between a focal point and an observed genotype). This score was used as argument in the

exponential kernel employed in the reproducing kernel Hilbert spaces model. The scoring was based on a procedure described by NEEDLEMAN and WUNSCH (1970), who used an algorithm to search for similarities in the amino acid sequence of two proteins. The procedure was modified and adapted to search similarities in a sequence of SNPs within chromosomes. The score was calculated using a dynamic programming algorithm described next.

Let \mathbf{x}_i and \mathbf{x}_j be two sequences of SNPs within a given chromosome, with the SNPs sorted in ascending order regarding their position in the chromosome. First, calculate the frequencies (f_{sk}) at locus s of genotype k (with $k = 1, 2$, or 3 being one of the three possible genotypes, *i.e.*, AA , Aa , or aa). Then, initialize the score $S = 0$. The algorithm for scoring similarity between two sequences i and j is computed from s going from 1 to the number of SNPs in a given chromosome as

$$\text{if } \begin{cases} \text{SNP}_{si} = \text{SNP}_{sj} \Rightarrow \text{subscore} = \text{subscore} \times f_{sk} \\ \text{SNP}_{si} \neq \text{SNP}_{sj} \Rightarrow S = S + \text{subscore}; \quad \text{subscore} = 1. \end{cases}$$

Above, S is the score for the similarity of two sequences of SNP in a given chromosome, and subscore is a temporary variable in the algorithm that is initialized to 1 at the beginning of the algorithm and every time the sequences differ in their genotypes. The final score between both sequences is given by the sum of the scores for each chromosome as

$$\text{Score}(\mathbf{x} - \mathbf{x}_i) = \sum_{\text{chr}=1}^{\text{no. chromosomes}} S_{\text{chr}}.$$

The lower the score, the more similar the SNP sequences are. Further, the more uncommon SNPs two sequences share, the lower the score is. This score was introduced in the exponential kernel as $K_h(\mathbf{x} - \mathbf{x}_i) = \exp[-\text{Score}(\mathbf{x} - \mathbf{x}_i)]$, so that the more similar two sequences were, the larger the value of the kernel between sequence \mathbf{x} and \mathbf{x}_i was.